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TÍTULO: ANALYSIS OF THE NON-GAUSSIAN SPECTRA
OF INTERPLANETARY SCINTILLATIONS

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ANALYSIS OF THE NON-GAUSSIAN SPECTRA
OF INTERPLANETARY SCINTILLATIONS

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ABSTRACT

The non-Gaussian intensity fluctuation spectra observed by Cohen, et al. (1967 a) are analysed. Computations of the length scales derived from the phase autocorrelation functions using Buckley's method (1971,1) indicate that for a r.m.s. phase deviation of 4 radians or more the diffracting medium behaves as one with its phase structure having "inner" and "outer" scales of turbulent blobs or eddies which are present in a turbulent medium.

1. INTRODUCTION

Interplanetary scintillations of small diameter radio sources were first observed by Hewish, Scott and Wills (1964). Using observations on several sources at 178 MHz they deduced the scale and velocity of the density irregularities in the solar wind and made estimates of the limits of diameters of the sources. Hewish and Okoye (1965) and Hewish, Dennison and Pilkington (1966) extended these observations to lower frequencies while Cohen (1965) extended them to higher frequencies.

Cohen, et al. (1967^a) computed certain statistical properties, such as the power spectra of radio scintillations, probability distribution of intensity, autocorrelation functions, scintillation index, etc., using observations on the sources 3C 138, 273, 287, 286 and 298 made with the 1000-foot radio telescope at the Arecibo Ionospheric Observatory, Puerto Rico. They showed that the quantitative interpretation of their observations gave a picture of the scattering region which compared well with the theory of scattering by a random phase-changing screen (Salpeter, 1967).

The scintillation spectra were near-Gaussian, with their width (or square root second moment) varying between 0.4 and 0.8 c/s, for sources observed in the weak-scattering region. In the strong-scattering region, however, very non-Gaussian spectra were observed below 0.3 a.u. on 3C 273 at 195 MHz, 430 MHz and 611 MHz, with the (square root) second moment of the scintillation spectra roughly proportional to the wavelength.

In order to look for the cause of the irregularities more information, than just the scale size, is essential. For example, the spaced receiver observations of Hewish, et al. (1966) were used to estimate the speed of the irregularities from the observed time delays. Assuming a Gaussian irregularity distribution of the form e^{-x^2/ℓ^2} , where x is the spacing of the receivers, they also found the pattern scale $\ell = 143 \pm 25$ km.

Similarly, it may be possible to derive more information about the scattering region using the non-Gaussian power spectra seen by Cohen, et al. (1967 a) on 3C 273 in the strong-scattering region. The results of the present analysis are interpreted in terms of some of the statistical properties of the intensity fluctuations of waves diffracted by a one-dimensional random phase-changing screen which imposes on the waves a r.m.s. phase deviation much greater than one radian (Buckley, 1971, I).

2. DATA AND ANALYSIS

Figure 1 shows the scintillation spectrum observed by Cohen, et al. (1967 a) on 3C 273 at 195 MHz, 430 MHz and 611 MHz. The vertical arrows on the abscissa show the fluctuation frequency at which the thermal noise was comparable with the scintillations. On the left side of the arrows

are shown the eighty percent confidence levels.

All the three cases of the power spectra were subjected to the method of least squares and the following exponential laws were deduced:

$$\left. \begin{array}{l} \text{Case I. } S_i(f) = 41.3 e^{-0.115 f} \\ \text{Case II. } S_i(f) = 40.0 e^{-0.205 f} \\ \text{Case III. } S_i(f) = 27.4 e^{-0.346 f} \end{array} \right\} \quad (1)$$

where Cases I, II, and III correspond to the spectra for the operating frequencies of 195 MHz, 430 MHz and 611 MHz respectively; $S_i(f)$ is the power density in decibels and f is the fluctuation frequency in c/s.

Using these power spectra of the intensity fluctuations it is possible to deduce, under some assumptions, the spatial distribution of the complex wave amplitude on emergence from the scattering region. Firstly, autocorrelation functions of the intensity patterns can be computed by converting the frequency spectra to spatial spectra which are the Fourier transforms of intensity autocorrelation functions. Assuming that the spatial irregularities are convected across the line of sight by the solar wind, the frequency spectra in (1) can be converted to spatial spectra by using the relation

$$f = \frac{V_s q}{2\pi}, \quad \text{cycles/sec.} \quad (2)$$

(Salpeter, 1967). Here V_s is the solar wind velocity in (meters/sec.) and q is the spatial wave number per meter.

Taking $V_s = 350$ km/sec as being characteristic of these observations, we obtain the three spatial spectra as

$$\left. \begin{array}{l} \text{Case I. } S_i(q) = 41.3 e^{-Aq} \\ \text{Case II. } S_i(q) = 40.0 e^{-Bq} \\ \text{Case III. } S_i(q) = 27.4 e^{-Cq} \end{array} \right\} \quad (3)$$

where $S_i(q)$ is the power density in decibels, $A = 6.4 \times 10^3$ (m),
 $B = 11.5 \times 10^3$ (m) and $C = 19.4 \times 10^3$ (m).

Secondly, the autocorrelation function of the intensity pattern is obtained by taking the Fourier transform of the spatial spectrum. Following Blackman and Tuckey (1959) the one-sided cosine transforms are given by

$$\rho_i(x) = 2 \int_0^{\infty} S_i(q) \cos(qx) dq \quad (4)$$

where $\rho_i(x)$ is the intensity autocorrelation function in the x-direction in a plane parallel to the scattering region.

Substituting in (4), we get

$$\text{Case I. } \rho_i(x) = 2 \int_0^{\infty} \cos(qx) 41.3 e^{-Aq} dq$$

$$= 2 (41.3) \frac{A}{A^2 + x^2}$$

$$\text{Case II. } \rho_i(x) = 2 (40.0) \frac{B}{B^2 + x^2}$$

$$\text{Case III. } \rho_i(x) = 2 (27.4) \frac{C}{C^2 + x^2} \quad (5)$$

Erdelyi, et al. (1954), where x is the distance in meters and A, B, C , have the values given in (3). These intensity autocorrelation functions (normalized to unity at the origin) are shown in Figure 2.

Now, Bramley (1951) has shown that if a complex field $f(x)$ has a randomly phased spectrum, then

$$\rho_{A^2}(r) = [\rho_f(r)]^2 \approx \rho_A(r) \quad (6)$$

where $\rho_A(r)$ and $\rho_{A^2}(r)$ are the autocorrelation functions of the amplitude and square of the amplitude respectively; while $\rho_f(r)$ is the autocorrelation function of the complex field $f(x)$. The probability distribution of the amplitude $A(x)$ in this case is Rayleigh [according to Cohen, et al. (1967 a) the probability distribution of fluctuation intensities was roughly Rice-square].

Again, Bramley (1954) has shown that the autocorrelation function $\rho_f(r)$ for the complex wave amplitude f at two points a distance r apart, along the screen, is given by

$$\rho_f(r) = \exp \left[-\phi_0^2 (1 - \rho_\phi(r)) \right] \quad (7)$$

where it is assumed that the phase change ϕ imposed on the wavefront is a random function of position on the screen, and ϕ is normally distributed over the screen, with r.m.s. value ϕ_0 ; $\rho_\phi(r)$ in (7) is the phase autocorrelation function.

Furthermore, although the complex amplitude changes with distance from the diffracting region, its autocorrelation function is independent of the distance and sufficiently defines the spatial distribution of the complex amplitude over a plane directly below the scattering region, (Ratcliffe, 1956).

Thus we can write

$$\rho_i(x) = [\rho_f(x)]^2 \quad (8)$$

From (7), we get

$$\rho_\phi(x) = 1 + \frac{1}{\phi_0^2} \ln \rho_f(x) \quad (9)$$

the phase autocorrelation function over the diffracting region.

Substituting for $\rho_f(x)$ using (5) and (8), we get for the three cases

$$\text{Case I. } \rho_{\phi}(x) = 1 + \frac{\ln k_1}{2\phi_0^2} - \frac{1}{2\phi_0^2} \ln(A^2 + x^2)$$

$$\text{Case II. } \rho_{\phi}(x) = 1 + \frac{\ln k_2}{2\phi_0^2} - \frac{1}{2\phi_0^2} \ln(B^2 + x^2) \quad (10)$$

$$\text{Case III. } \rho_{\phi}(x) = 1 + \frac{\ln k_3}{2\phi_0^2} - \frac{1}{2\phi_0^2} \ln(C^2 + x^2)$$

where $k_1 = 2 A (41.3) \text{ m}^2$, $k_2 = 2 B (40.0) \text{ m}^2$, and $k_3 = 2 C (27.4) \text{ m}^2$.

The phase autocorrelation function was computed in all the three cases for values of ϕ_0 from 1 to 10 radians. Figure 3 shows these functions (normalized to unity at the origin) for the case where $\phi_0 = 2 \text{ rad}$.

Now, as ϕ_0 increases further $\rho_{\phi}(x)$ approaches zero more slowly as $x \rightarrow \infty$. For $\phi_0 > 2 \text{ rad}$, it will be interesting to see how the diffracting region behaves. For this situation, Buckley (1971, I) has given a method which is essentially that of steepest descent. The structure of the one-dimensional diffracting screen is specified by several length scales which are derived from the phase autocorrelation function. It is shown that if the function has a single implicit length scale, such as $\exp(-x^2/\ell^2)$, these scales are comparable with each other. If, however, $\rho_{\phi}(x)$ has two implicit length scales, having magnitudes very much different from each other, then they correspond to the "inner" and "outer" scales of turbulent blobs or eddies which would result from a turbulent medium.

Buckley (1971, I) has characterized the function $\rho_\phi(x)$ by means of the following length scales:

$$\left. \begin{aligned} L_0 &= (1/2\pi) \int_{-\infty}^{\infty} \rho_\phi(x) dx \\ L_2 &= [-\rho_\phi^2(0)]^{-1/2} \\ L_4 &= [\rho_\phi^4(0)]^{-1/4} \\ L_6 &= [-\rho_\phi^6(0)]^{-1/6} \end{aligned} \right\} \quad (11)$$

where

$$\rho_\phi^n(x) = d^n \rho_\phi / dx^n.$$

For values of $\phi_0 > 2$ rad., for which this method is valid, we calculated these length scales for all the three spectra and found that when $\phi_0 \geq 4$ rad. the outer scale L_0 starts assuming values much larger than the inner scale L_6 , or even the intermediate scales L_2 and L_4 .

The results of these calculations are summerized in Table I for $\phi_0 = 4$ rad. Values of L_0 were computed taking the correlation length to correspond to the point where $\rho_\phi(x)$ drops down to $1/e$ of its initial value.

TABLE I

Length scales of the phase autocorrelation function for $\phi_0 = 4$ rad.

Operating Frequency of Observed Spectra MHz	Length Scales of $\rho_\phi(x)$ km			
	L_0	L_2	L_4	L_6
195	-910	25.6	8.2	4.6
430	-2×10^4	46	14.7	8.2
611	-3×10^4	77.6	24.9	13.6

From Table I it is seen that in all the three cases of the observed spectra the following inequality holds good:

$$L_0 \gg L_2 > L_4 > L_6 \quad (12)$$

as discussed by Buckley (1971, I) in the case of a turbulent region.

Now if, as he has shown, L_0/ℓ is a large number then approximately

$$\frac{L_0}{L} \sim 1, \quad \frac{L_2}{L} \sim \left[\frac{\ell}{L}\right]^{(3-\gamma)/2} \quad (13)$$

where ℓ and L are respectively the "inner" and "outer" scales of $\rho_\phi(x)$ and γ is the power index of a highly idealized turbulence or Kolmogorov spectrum

$$S_\phi(q) = L_0 (qL)^{-\gamma}$$

with the power spectrum $S_\phi(q)$ specifying the phase correlation.

From (13) we can write

$$\frac{L_2}{L_0} = \left(\frac{L_6}{L_0} \right)^{(3-\gamma)/2} \quad (14)$$

Substituting the values of the length scales from Table I γ can be estimated. Table II shows the results:

TABLE II

Values of the power index γ .

Operating Frequency of Observed Spectra MHz	Power Index γ
195	1.65
430	1.45
611	1.45

Thus γ lies between about 3/2 and 5/3 which is true for a Kolmogorov spectrum (Tatarski, 1961).

Again, using these values of the length scales it is possible to estimate the scale of the diffraction pattern of intensity far from the diffracting region. Buckley defines this scale as

$$L_p = [-\rho_i^2(0)]^{-1/2} \quad \text{or} \quad L_p^{-2} = \int_{-\infty}^{\infty} q^2 S_i(q) dq$$

where L_p corresponds to the L_2 scale of the phase pattern. He has shown that when the screen distance approaches infinity

$$L_p = \frac{L_2}{\sqrt{2}\phi_0}.$$

Substituting values of L_2 from Table I we get

$$\begin{aligned} L_p &\approx 5 \text{ km} && \text{for } 195 \text{ MHz} \\ &\approx 9 \text{ km} && \text{for } 430 \text{ MHz} \\ &\approx 15 \text{ km} && \text{for } 611 \text{ MHz} \end{aligned}$$

Thus the scale of the diffracting pattern is comparable with the inner (smaller) scale of the phase pattern. According to Buckley (1971,1) this is true if the phase autocorrelation function has two scales.

3. CONCLUSIONS

Cohen, et al. (1967a) have discussed their observations of the non-Gaussian fluctuation spectra on 3C 273 in terms of a diffraction regime diagram based on a summary of the theory of diffraction of radio waves on emergence from the medium (Salpeter, 1967). According to them these observations belonged to the strong-scattering Regions III and IV. If this interpretation is valid, then the exponential spectra can be used to investigate the structure of the scattering region with the help of the method of steepest descent which Buckley (1971, I) applied to diffraction by a random phase-changing screen with very large r.m.s. phase deviation. At $\phi_0 = 4$ rad., for which the computations were made, the observations would pertain to the elongation of the source $\epsilon < 8^\circ$ for 195 MHz and $\epsilon < 6^\circ$ for 430 MHz and 611 MHz. This conclusion agrees well with the observations of Cohen, et al. (1967 a).

The present analysis has shown that for $\phi \geq 4$ rad. the diffracting region comprises a phase structure with inner and outer scales, similar to those in a highly turbulent region of the Kolmogorov type in which the inner and outer scales represent respectively the viscous dissipation length, with negligible energy below, and the length of the "eddies" which contain most of the energy of the turbulence.

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FIGURE CAPTIONS

- Figure 1: Scintillation spectra observed by Cohen, et al. (1967 a) on 3C 273 at 195, 430 and 611 MHz.
- Figure 2: Intensity autocorrelation functions derived from the three spectra.
- Figure 3: Phase autocorrelation functions for the three spectra.

FIG. 1

FIG. 2

FIG. 3