1. Publication Nº <i>INPE-3903-PRE/944</i>	2. Version <i>2nd.</i>	3. Date <i>May., 1986</i>	5. Distribution ☐ Internal ☒ External ☐ Restricted
4. Origin <i>DMC/DGC</i>		Program <i>CONTAT</i>	
6. Key words - selected by the author(s) <i>STATE ESTIMATION OPTIMAL CONTROL</i> <i>ADAPTIVE FILTERING</i> <i>KALMAN FILTERING</i>			
7. U.D.C.: <i>681.5.015.44</i>			
8. Title <i>USE OF A VIRTUAL CONTROL APPROACH IN STATE ESTIMATION</i>		10. Nº of pages: <i>7</i>	
		11. Last page: <i>6</i>	
9. Authorship <i>A.T.Fleury</i> <i>A.Rios-Neto</i>		12. Revised by <i>Helio Koiti Kuga</i>	
Responsible author		13. Authorized by <i>Marco Antonio Raupp</i> <i>Director General</i>	
14. Abstract/Notes <i>This work presents the development of a state estimator for dynamic systems exploring the dual function existing between state estimation and optimal control problems. An estimator algorithm similar to the Extended Kalman Filter results to be used in real-time nonlinear systems which can recursively be approximated by linear systems. The original estimation problem is transformed into an equivalent one of virtual control. This control problem is then used to generate and adaptive, locally convergent algorithm where, instead of full state estimation, one has to estimate just a control vector with smaller dimension than the state vector. The control formulation also allows the development of convergence acceleration criteria. Two of these criteria are tested in this exploratory phase and others shall be tested and presented in the final version of the paper. The algorithm is implemented in a digital computer to estimate the orbit of a low Earth orbit satellite under simulated conditions. Numerical results are presented for a test case considering critical initial values for the estimator and observations provided by three Earth stations.</i>			
15. Remarks <i>2nd version - 19/Feb/87</i> <i>Draft paper, submitted for presentation in the 10th World Congress of IFAC.</i>			

Typing the sheet
with the use of
of page 11 should
be indicated here

THE LAYOUT FOR
OF YOUR PAPER
SHOULD BE AS
INDICATED HERE

USE OF A VIRTUAL CONTROL APPROACH IN STATE ESTIMATION

A.T. Fleury and A. Rios-Neto
Instituto de Pesquisas Espaciais - INPE
Ministério de Ciência e Tecnologia - MCT
Av. dos Astronautas, 1758
Mail Box 515
12201 - São José dos Campos - SP - Brazil

Abstract: This work presents the development of a state estimator for dynamic systems exploiting the dual function existing between state estimation and optimal control problem. An estimator algorithm similar to the Extended Kalman Filter results to be used in real-time nonlinear systems which can recursively be approximated by linear systems. The original estimation problem is transformed into an equivalent one of virtual control. This control problem is then used to generate and adaptive, locally convergent algorithm where, instead of full state estimation, one has to estimate just a control vector. The control formulation allows different control structures and different convergence acceleration criteria. Two control structures and three criteria are tested in this exploratory phase. The algorithm is implemented in a digital computer to estimate the orbit of a low Earth orbit satellite under simulated conditions. Numerical results are presented for a test case considering critical initial values for the estimator and observations by three Earth stations.

Keywords: State estimation, Adaptive filtering, Kalman filtering, Optimal control.

1. INTRODUCTION

The state estimation of a very common problem in dynamic systems represents a very common problem in state estimation of multivariable systems. In this case, the dynamic model adopted for the estimator is only a crude approximation of reality and, as a consequence, unmodelled effects, mostly for non-linear systems, can cause divergence of the estimator. In order to avoid divergence, error compensation techniques are usually employed with different versions of the Kalman Filter (e.g., Maybeck, 1979; Jazwinski, 1970; Gelb et al., 1974). These techniques generally explore the information given by the observation residues to either directly estimate the unmodelled effects (e.g., Tapley and Ingram, 1977; Cruz and Rios Neto, 1980; Rios Neto and Cruz, 1985) or to condition the state error covariance matrix to keep the capability of the estimator to extract information from the new observations (e.g., Jazwinski, 1969; Rios Neto and Kuga, 1981, 1982, 1985).

This exploratory work presents a new alternative scheme for the state estimation problem, specially for nonlinear systems. The duality between the functions of estimation and optimal control is explored to transform the original estimation problem into one of tracking the observations with a virtual control. The key idea is that, independent of the dynamic model for the estimator, if the system is completely controllable and observable, one can choose a desirable control action to drive the system towards a region defined by the observations in a finite time interval. Without the need of increasing the number of estimated variables, the observation residues are used to estimate the virtual control necessary to update the estimate of the state. Besides that, the virtual nature of the tracking control allows the possibility of imposing conditions of controllability to better extract the information contained in the observations (Rios Neto and Fleury, 1984; 1985; Fleury, 1985). Therefore, the proposed

estimator is indicated for situations where there is a great lack of knowledge about the system dynamics, the observations, preliminary tests of the proposed estimator were done under digitally simulated conditions for a critical problem of real-time orbit estimation of a low altitude artificial satellite. Results are shown to be satisfactory for this exploratory phase of procedure.

The problem to be solved is the state estimation of a multivariable dynamic system of the type:

$$\dot{x} = f(x, t) + E^T(x, t) + G(t)w(t), \quad (1)$$
$$y(t_k) = h_k(x(t_k), t_k) + v(t_k), \quad k = 1, 2, \dots, (2)$$
where x is the $n \times 1$ state vector; $w(t)$ and $v(t_k)$ are $m \times 1$ and $r \times 1$ zero mean independent Gaussian white noises with the usual hypothesis of $w(t)$ being independent of the past states and $v(t_k)$ being independent of the state, with covariances:
$$E[w(t)w^T(\tau)] = \eta(t)\delta(t-\tau),$$
$$E[v(t_k)v^T(t_j)] = R(t_k)\delta_{kj}, \quad (3)$$
where $\delta(t-\tau)$ is the Dirac delta function and δ_{kj} is the Kronecker symbol. In this problem, the term $h_k(x, t)$ in equation (1) represents the unknown part of the dynamical model, which usually cannot be included in the estimator model because of lack of knowledge about the system dynamics.

Consider now a typical discretization interval, (t_k, t_{k+1}) , in the prediction phase of an Extended Kalman Filter (e.g., Jazwinski, 1970), a nominal trajectory is generated by:

DO NOT TYPE OUTSIDE THIS AREA

DO NOT TYPE OUTSIDE THIS AREA

From the all pages
with the exception of
page 11 should
be removed here

THE LAYOUT FOR
THE FIRST PAGE
OF YOUR PAPER
SHOULD BE AS
INDICATED HERE

$$\hat{x} = f(\hat{x}, t) ; \hat{x}(t_k) = \hat{x}(t_k | t_k) \quad (4)$$

where $\hat{x}(t_k | t_k)$ is the estimate in t_k . Linearizing the system given in Equation (1) around the nominal trajectory and defining:

$$\Delta \hat{x}(t) = \hat{x}(t) - \hat{x}(t) \quad (5)$$

one obtains a first order approximation of the propagated error as given by:

$$\Delta \hat{x}(t_{k+1}) = \phi(t_{k+1}, t_k) \Delta \hat{x}(t_k) +$$

$$\int_{t_k}^{t_{k+1}} \phi(t_{k+1}, s) \bar{G}(s) w(s) ds. \quad (6)$$

In Equation (6), $\phi(\cdot, \cdot)$ is the state transition matrix associated with the linear system:

$$\dot{\Delta \hat{x}}(t) = \bar{F}_x(\bar{x}, t) \Delta \hat{x}(t) + \bar{G}(t) w(t), \quad (7)$$

where the subindex x indicates a partial derivation with respect to the state. The propagated error can be regarded as a priori information in t_{k+1} , which is the information based on previously processed observations:

$$\Delta \hat{x}(t_{k+1}) = \Delta \hat{x}(t_k) + \eta(t_{k+1} | t_k). \quad (8)$$

Since the nominal trajectory was taken with the initial value in t_k equal to the estimated value at that time, then the propagated estimate in t_{k+1} is necessarily zero. Therefore, if the approximations of Equations (4) and (6) are assumed, there results:

$$\Delta \hat{x}(t_{k+1}) = 0 + \eta(t_{k+1} | t_k), \quad (9)$$

where $\eta(t_{k+1} | t_k)$ is zero mean conditioned on the observations already processed with covariances given by:

$$E[\eta(t_{k+1} | t_k) \eta^T(t_{k+1} | t_k)] \Delta P(t_{k+1} | t_k) = \phi(t_{k+1}, t_k) P(t_k | t_k) \phi^T(t_{k+1}, t_k) + \int_{t_k}^{t_{k+1}} \phi(t_{k+1}, s) \bar{G}(s) \bar{Q}(s) \bar{G}^T(s) ds. \quad (10)$$

In the proposed procedure the state estimation of the system in Equation (1) is approximated by the estimation of a "virtual control". Defining:

$$\bar{x}^c = f(\bar{x}^c, t) + \bar{G}(t) u(t), \quad (11)$$

the control vector $u(t)$ is to be estimated in order to force the controlled state $\bar{x}^c$ to be a good approximation of the true state x by tracking the observations given by Equation (2). From an heuristic point of view, the idea is quite simple: Consider a typical interval (t_k, t_{k+1}) . Since the model of the estimator is not a good approximation to the real system, within the interval t_k to t_{k+1} , the propagated trajectory will deviate relative to the true one. A control action, based on the observation residuals, is then calculated in t_{k+1} to update the estimate of the state vector. This action changes the initial condition for the nominal trajectory in t_{k+1} to a point closer to the

Please do not fold this sheet

70% A4

DO NOT TYPE OUTSIDE THIS AREA

true trajectory than the original one. In some sense, this procedure resembles the extended Kalman filter, but it must be pointed out that the virtual control scheme requires a number of estimated degrees of freedom of the system (control-stability), and also requires a number of observations with enough level of information which guarantees the calculation of the control action in t_{k+1} (observability).

To obtain, in a first approach, the control $u(t)$ in the interval t_k to t_{k+1} , one assumes u as a first order perturbation and takes the controlled trajectory as:

$$\bar{x}^c = f(\bar{x}^c, t), \quad \bar{x}^c(t_k) = \hat{x}(t_k | t_k). \quad (12)$$

Using the same steps given by Equations (5) and (6):

$$\Delta \hat{x}^c = \bar{F}_x(\bar{x}^c, t) \Delta \hat{x}^c + \bar{G}(t) u(t), \Delta \hat{x}^c(t_k) = 0, \quad (13)$$

where $u(t)$ is modelled as a step process and calculated to satisfy:

$$\Delta \hat{x}^c(t_{k+1}) = \Delta \hat{x}(t_{k+1}), \quad (14)$$

$$y(t_{k+1}) = h_{k+1}(\bar{x}^c(t_{k+1}) + \Delta \hat{x}^c(t_{k+1}), t_{k+1}) +$$

$$v(t_{k+1}). \quad (15)$$

$$0 = \Delta \hat{x}^c(t_{k+1}) + \eta(t_{k+1} | t_k). \quad (16)$$

But from Equation (13) one obtains:

$$\Delta \hat{x}^c(t_{k+1}) = \left(\int_{t_k}^{t_{k+1}} \phi(t_{k+1}, s) \bar{G}(s) \bar{G}^T(s) ds \right) \Delta \hat{x}^c(t_k) +$$

$$\Delta y(t_{k+1}) = \left[\frac{\partial h}{\partial \bar{x}^c} h_{k+1}(\bar{x}^c(t_{k+1}), t_{k+1}) \right] \Delta \hat{x}^c(t_{k+1}) +$$

$$v(t_{k+1}). \quad (17)$$

where $\phi(t, t_k)$ is the transition matrix associated with Equation (13). From the linearization of Equation (15) one gets:

$$\Delta y(t_{k+1}) = \left[\frac{\partial h}{\partial \bar{x}^c} h_{k+1}(\bar{x}^c(t_{k+1}), t_{k+1}) \right] \Delta \hat{x}^c(t_{k+1}) +$$

$$v(t_{k+1}). \quad (18)$$

where high order terms have been disregarded. Finally, combining the results of Equations (16), (17) and (18), the following problem of parameter estimation results:

$$0 = y(t_{k+1}, k) u(t_k) + \eta(t_{k+1} | t_k), \quad (19)$$

$$\Delta y(t_{k+1}) = h(t_{k+1}, k) u(t_k) + v(t_{k+1}), \quad (20)$$

$$H(k+1) \Delta \frac{\partial x}{\partial \bar{x}^c} h_{k+1}(\bar{x}^c(t_{k+1}), t_{k+1}). \quad (21)$$

Using a Gauss-Markov minimum variance estimator (e.g., Liebelt, 1967 and Maybeck, 1979), estimates of $u(t_k)$ are obtained:

$$u(t_k) = P^n(k+1, k) y^T(k+1, k) H^T(k+1) R^{-1}(k+1) y(t_{k+1}) \quad (22)$$

The idea of restoration was first used in numerical gradient-like methods to solve optimal control problems. In this case, restoration is used to modify C_p and C_v in Equation (30), thus giving priority of convergence to either forced (velocity type) state variables or non forced (position

3.2. RESTORATION

where C_p and C_v are positive coefficients, held fixed during the time propagation (Rios Neto and Fleury, 1984). Despite the advantage given by the simple form, this correction technique has the drawbacks of using a trial-and-error method to choose C_p , C_v and of adding the estimated corrections to the state variables with the same sign.

$$G^T \Delta [C_p I_{n/2} : C_v I_{n/2}] \quad (30)$$

The simplest form to define a control matrix with the characteristic described before is to consider:

3.1. DIRECT CORRECTION

For the control matrix G are proposed. and state variables and three types of corrections augment the coupling between estimated controls the control matrix G in Equation (11) in order to virtual control context it is possible to define ling between state variables and noises. In the that the coefficients in G just make the coup- matrices of order $n/2$, respectively. This means where $0_{n/2}$ and $I_{n/2}$ are the null and the identity

$$G^T \Delta [0_{n/2} : I_{n/2}] \quad (29)$$

$$x = f(x,t) + G w(t) \quad (28)$$

It is usual to have the matrix G in: governed by second-order differential equations, estimation of mechanical systems or other systems ally, when one is faced with the problem of state of the estimator dynamical model structure. General mutation allows to get some advantages in terms In the proposed procedure, the virtual control for

3. CONTROL MATRIX CORRECTION

$$P(k+1, k) = P(k, k) + Y^T(k, k) Y(k, k) \quad (27)$$

$$P(k+1, k+1) = P(k, k) + E [e(k, k) e^T(k, k)] \quad (26)$$

$$e(k, k+1) = x(k, k+1) - \hat{x}(k, k+1) = \Delta x - \Delta \hat{x} \quad (25)$$

To recover the covariance matrix of the error in the estimate, it is only necessary to consider Equations (14), (17) and (25) to get:

$$- \hat{x}(k, k+1) | e(k, k+1) = \hat{x}(k, k+1) + \Delta \hat{x}(k, k+1) \quad (24)$$

and to take the approximation:

$$\Delta \hat{x}(k, k+1) = Y(k, k+1) \Delta u(k, k) \quad (23)$$

To recover the estimates of the state, one shall combine Equations (17) and (22) to get:

$$\text{where } R(k+1) \text{ and } P(k+1, k) \text{ are as defined in Equations (3) and (10).}$$

$$P(k+1, k) = [Y^T(k, k+1) P^{-1}(k, k) Y(k, k+1) + R(k+1)]^{-1} \quad (22)$$

type) state variables, depending on the system behavior. A decision to change C_p and C_v is made based upon a convergence measure given by estimated errors, e_p for nonforced variables and e_v for the forced ones defined by:

$$e_v(k) = \frac{1}{n} \sum_{i=1}^n \frac{1}{2} P_{ii}^{1/2}(k/k) \quad (31)$$

$$e_p(k) = \frac{1}{n/2} \sum_{i=1}^{n/2} \frac{1}{2} P_{ii}^{1/2}(k/k) \quad (32)$$

$$\Delta x(k+1, k) = \phi(k+1, k) \Delta x(k, k) + B(k, k) u(k, k) \quad (34)$$

$$J = [\Delta y(k, k+1) - H(k+1, k) \Delta x(k, k+1)]^T [\Delta y(k, k+1) - H(k+1, k) \Delta x(k, k+1)] \quad (35)$$

$$\Delta x(k+1, k) = \phi(k+1, k) \Delta x(k, k) + B(k, k) u(k, k) \quad (36)$$

$$J = [\Delta y(k, k+1) - H(k+1, k) \Delta x(k, k+1)]^T [\Delta y(k, k+1) - H(k+1, k) \Delta x(k, k+1)] \quad (37)$$

$$\Delta x(k+1, k) = \phi(k+1, k) \Delta x(k, k) + B(k, k) u(k, k) \quad (38)$$

$$J = [\Delta y(k, k+1) - H(k+1, k) \Delta x(k, k+1)]^T [\Delta y(k, k+1) - H(k+1, k) \Delta x(k, k+1)] \quad (39)$$

$$\Delta x(k+1, k) = \phi(k+1, k) \Delta x(k, k) + B(k, k) u(k, k) \quad (40)$$

DO NOT TYPE OUTSIDE THIS AREA

DO NOT TYPE OUTSIDE THIS AREA

DO NOT TYPE OUTSIDE THIS AREA

DO NOT TYPE OUTSIDE THIS AREA

THE LAYOUT FOR THE FIRST PAGE OF YOUR PAPER SHOULD BE AS INDICATED HERE

Fig. 1. For all pages of paper, it should contain the same information

Please do not fold this sheet

70%

THE LAYOUT FOR
THE FIRST PAGE
OF YOUR PAPER
SHOULD BE AS
INDICATED HERE

convergence; (3.0;1.0) for convergence in position
and velocity; and (0.1;2.0) for poor velocity con-
vergence (see Section 3.2).

Fig. 1. True and estimated position errors: direct correction, ASNE

Fig. 2. True and estimated velocity errors: direct correction, ASNE

Fig. 3. True and estimated position errors: restoration, CLN

Fig. 4. True and estimated velocity errors: restoration, CLN

Figures 5 and 6 present the results accomplished with the proposed procedure aided by the automatic Correction and the Adaptive State Noise Estimation technique. Figures 7 and 8 show the results obtained with the use of the Automatic Correction with constant level noise addition. The results obtained using State Error Feedback and constant level noise addition in the test problems were very similar to those obtained with the Direct Criterion and are not shown. Coefficient β was chosen equal to 10^{-4} . Direct correction was also used with C_p and C_v equal to 3.0 and 1.0, respectively.

Fig. 5. True and estimated position errors: automatic correction, ASNE

Fig. 6. True and estimated velocity errors: automatic correction, ASNE

Fig. 7. True and estimated position errors: automatic correction, CLN

Fig. 8. True and estimated velocity errors: automatic correction, CLN

The analysis of Figures 1 to 8 shows that good results are accomplished with the use of the Automatic Correction and satisfactory convergence is obtained with the other techniques. In these cases the estimated errors in the transient phase of convergence are optimistic when compared to the real ones, characterizing the difficulty in introducing adequate noise levels. The time interval for convergence in this transient phase is quite long, and a comparison shows that convergence for positions is better in the case of the Restoration procedure. These characteristics determine that other improvements must be done in the technique in order to reach a better performance. On the other hand, results obtained with the Automatic

DO NOT TYPE OUTSIDE THIS AREA

DO NOT TYPE OUTSIDE THIS AREA

Correction scheme, both with the Adaptive State Noise Estimation technique and with the Constant Level Noise Addition, are comparable to results presented by other techniques usually employed in nonlinear system estimation, such as the Extended Kalman Filter aided by Dynamic Model Compensation (e.g., Kuga, 1982). In this case, a much faster convergence is obtained at the cost of an increment of around 20% in computer processing time, when compared to the other techniques.

6. CONCLUSIONS

A new approach to state estimation of nonlinear systems has been presented, where the duality between control and estimation was explored to transform the estimation problem into one of determining a virtual control. As indicated by the results obtained in this exploratory phase, it is expected to be a valid alternative for the case where a great lack of knowledge in the dynamic exists, but a good level of information is locally provided by the observations.

Up to this point, one only started to explore the possibilities opened by the virtual nature of control in Equation (11). Presently, efforts are being done to further explore these possibilities in order to improve the different estimator versions performance.

REFERENCES

CRUZ, J.J.; RIOS NETO, A. (1980). Estimating the State of Ships in the Presence of Unmodelled Accelerations. American Tank Conference, 19, Ann Arbor, Michigan, USA.

FLURY, A.T. (1985). Estimadores de Estado de Qualidade Dinâmica. São Paulo, SP: Editora da Universidade de São Paulo, 1985.

CELE, A.; KASPER JR., J.P.; NASH JR., R.A.; PRICE, C.F.; SUTHERLAND JR., A.A. (1974). Applied Optimal Estimation. Cambridge, MA, MIT Press.

JAZWINSKI, A.H. (1969). Adaptive Filtering. Automatica, 5:475-485.

JAZWINSKI, A.H. (1970). Stochastic Processes and Filtering Theory. New York, Academic Press.

KUGA, H.K. (1982). Estimador Adaptativo de Órbita (Adaptive Orbit Estimator Applied to Low Altitude Satellites) MSc Thesis (Space Sciences). São José dos Campos, INPE, (INPE-2316-TDL/079).

LIEBELT, P.B. (1967). An Introduction to Optimal Estimation. Menlo Park, CA, Addison Wesley.

MAYNECK, P.S. (1979). Stochastic Models, Estimation and Control. V.1, New York, Academic Press.

(Mathematics in Science and Engineering, 141-1).

RIOS NETO, A.; CRUZ, J.J. (1985). A Stochastic Rudder Control Law for Ship Path-Following Autopilots. Automatica, 21(4): 371-384.

RIOS NETO, A.; FLURY, A.T. (1984). Utilização de um Estimador de Estado (Use of the Duality Concept in the Development of a State Estimator). São José dos Campos, INPE, (INPE-3288-RP/110).

RIOS NETO, A.; FLURY, A.T. (1985). State Estimation Procedure Based Upon the Duality Concept. Presented at the 2nd IASTED International Conference in Telecommunication and Control, Rio de Janeiro.

RIOS NETO, A.; KUGA, H.K. (1981). Estimador Adaptativo e em Tempo Real de Órbita de Satélites Artificiais a Baixa Altitude (Real-Time Adaptive Orbit Estimation of Artificial Satellites Artificiais a Baixa Altitude).

DO NOT TYPE OUTSIDE THIS AREA

THE LAYOUT FOR THE FIRST PAGE OF YOUR PAPER SHOULD BE AS INDICATED HERE

PLEASE DO NOT TYPED THIS AREA

Satellites at Low Altitude). In: Sixth Brazilian Congress of Mechanical Engineering, Rio de Janeiro. Proceedings, ABEM, v.B., p. 405-414.

RIOS NETO, A.; KUGA, H.K. (1985). Kalman Filter-aided State Noise Adaptive Estimation. Presented at the 2nd, IASTED International Conference in Telecommunication and Control, Rio de Janeiro.

TAPLEY, B.D.; INGRAM, D.S. (1973). Orbit Determination in the Presence of Unmodelled Accelerations. IEEE Transactions Automatic Control, AC-18(4):369-373.

DO NOT TYPE OUTSIDE THIS AREA

DO NOT TYPE OUTSIDE THIS AREA