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RESUMO - NOTAS / ABSTRACT - NOTES <p>An interface for giving suggestions about how to modify weighting matrixes which appear in the formulation of optimal control problems with quadratic performance index is described. The suggestions come both from heuristics for choosing the matrix elements, some of them being obtained through simulations for several systems, and also from criteria for improving the system in relation to the specification of some parameters.</p>				
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AN INTELLIGENT INTERFACE FOR THE DESIGN OF OPTIMAL CONTROL SYSTEMS

R.Acerbi*, P.O.Simoni*, E.Cardoso**

(*) Instituto de Pesquisas Espaciais - INPE

(**) Instituto Tecnológico de Aeronáutica - ITA
São José dos Campos - SP - Brazil

ABSTRACT

An interface for giving suggestions about how to modify weighting matrixes which appear in the formulation of optimal control problems with quadratic performance index is described. The suggestions come both from heuristics for choosing the matrix elements, some of them being obtained through simulations for several systems, and also from criteria for improving the system in relation to the specification of some parameters.

1. INTRODUCTION

This paper describes an interactive system for aiding in the design of linearized time-invariant feedback control systems, which are described by state space equations of the form

$$\begin{aligned}\dot{\underline{x}}(t) &= \underline{A}\underline{x}(t) + \underline{B}\underline{u}(t). \\ \underline{y}(t) &= \underline{C}\underline{x}(t).\end{aligned}$$

The objective is to find a control law which satisfies a set of constraints

$$l_1(k,s) \leq p(k,s) \leq u_1(k,s),$$

where $p(k,s)$ is a parameter (time delay, time rising, time settling, overshoot,...) which may be specified by the designer for any state s , being $l_1(k,s)$ and $u_1(k,s)$ the lower and upper limits.

The system must also minimize a quadratic functional by finding an optimal $\underline{u}^*(t)$

$$J = \int_0^{\infty} [\underline{x}^T \underline{Q} \underline{x} + \underline{u}^T \underline{P} \underline{u}] dt,$$

where $\underline{Q}$ and $\underline{R}$ are weighting matrixes. The $\underline{A}$, $\underline{B}$, and $\underline{C}$ matrixes are fixed a priori and the parameters may be modified by suitable choices of the weighting matrixes.

These matrixes may be chosen either from past experience, theoretical studies for some particular cases, if available, or by simulation studies.

When dealing with single input systems, the phase-variable canonical form may be used:

$$\dot{\underline{x}}(t) = \underline{A}'\underline{x}(t) + \underline{b}'\underline{u}(t),$$

$$\underline{A}' = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix}; \quad \underline{b}' = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

where a_i are the coefficients of the characteristic polynomial of the system:

$$\sum_{i=0}^n a_i s^i, \text{ with } a_n = 1.$$

(the formulation of the optimal control problem may be found for instance in [3])

The current system version is constrained to the time domain analysis.

It is supposed that the system under design is completely controllable; the stability analysis is not needed. This implies $x_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for all i in $[1, n]$.

Some hypotheses are also supposed to hold for the matrixes. Matrix R is supposed to be positive definite and Q nonnegative definite (when J is minimized it is not the case that high values for each integrand occur with opposed sign and almost canceling themselves). Matrix R is also supposed to be diagonal. Matrix C is taken as the identity matrix.

In order to find the optimal $u^*(t)$ which minimizes the quadratic performance index J , the Riccati matrix equation

$$A^T P + PA + Q - PB^T R^{-1} BP = 0,$$

must be solved for the Riccati matrix P .

The interactive tool is intended to help a designer by:

a) using a set of rules of thumb concerning the choice of the elements of the Q or R matrixes to be modified;

b) performing a series of simulations of the system under design, in a range of values around the current q_{ij} or r_{ij} , in order to compute the parameters for each of such matrix elements under consideration;

c) analysing the simulation results and suggesting the more suitable update in Q or R , according to the number of parameters that stay out the specification intervals as well as how far the parameters are from the interval limits.

2. SIMULATION RESULTS AND OTHER HEURISTICS

The just mentioned rules of thumb were obtained by a set of simulations of systems written in canonical form (the matrix A is such that its last line corresponds to the system characteristic polynomial).

These rules were collected for various system classes, and, for each class, for many characteristic polynomials. For a given unsatisfied parameter, the rules give a suggestion about which element of Q or R matrixes may be changed in order to increase or decrease the parameter, but causing little variation in the satisfied ones. These heuristics avoid an exhaustive search over all matrix elements. Other rules of thumb were obtained from other publications or from experienced designers.

Simulations were performed initially over a set of systems with real and complex poles. The classes are given by the number and relative position of poles and zeros: only poles, conjugate poles, poles and one zero, etc.

The parameters under study had their behavior determined as a function of Q and R elements. For each system (given by A , B , and C), an initial pair of matrixes Q and R was chosen ($q_{ij} = 1$ for all i and j ; $R = I$). Each matrix element was considered at a time, varying in an interval around the initial value. The parameters were measured in each situation, for every state $x_1, \dots, x_n$.

Fig.1 : Parameter definitions

The evidences collected from the simulation studies were classified in the following way: if they suggest an increasing in the parameter value as a consequence of an increasing in a certain q_{ij} or r_{ij} ; if they suggest a decreasing, or if the parameter presented an oscillatory behavior when a matrix element was modified.

The chosen parameters

t_d - time delay

t_r - time rising

m_p - overshoot

t_p - time overshoot

are defined as in Figure 1, according to the class of temporal response.

Some of the curves of the parameter values against the q_{ij} values are depicted in Figure 2, for a third order system.

From these curves evidences were collected about the way matrix elements have influence over the parameters. These evidences were grouped in the form of matrixes like

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

for t_d in third order systems, where 1 means that an increasing in q_{ij} suggests an increasing in t_d , -1 the inverse suggestion, and 0 means that no suggestion can be done.

Fig.2: $p(k,1)$ versus q_{ij}

Sensitivity analysis was also performed for each parameter, in order to determine to what matrix element the parameter is more sensitive. Some of the joint analysis are shown in Figure 3.

A number of heuristics were derived from these simulation studies, to be used whenever there is at least one parameter out of the specification interval. Their effect can be summarized by a description of their

purpose: they allow reducing the search space through the sensibility analysis, and they allow deriving sensibility trees for the cases where there are more than one parameter out of the interval.

Fig.3: Analysis for q_{00}

Although there is some belief that the chosen parameters are correlated, the simulation studies have shown that they can exhibit a distinct behavior when Q and R elements are modified.

3. SYSTEM ARCHITECTURE

The proposed architecture is depicted in Figure 4. The following modules are defined:

- knowledge storage
- rules for interaction between designer and the system (R1)
- system definition (parameter specification)
- system solution
- rules for controlling the process and the design steps (R2)
- rules for interfacing the external routines (R3)

There is some resemblance between this architecture and that one used in [5]. The nature of the problem imposes however a heavy use of external routines to the environment for the

rules, in order to simulate the system under design.

The following steps can be identified in the design process:

- a) initialization: input of matrixes A , B , initial Q and R , parameter interval specifications
- b) test of Q and R for positive definiteness
- c) initial simulation for the current Q and R
- d) suggestion about which matrix elements should be modified
- e) additional simulations, one for each matrix element under consideration (suggested by the heuristics)
- f) computation of derivative approximations of the parameters in relation to each matrix element being considered
- g) computation of an estimated interval in which each matrix element can be increased or decreased, in order to decrease or maintain the number of parameters out of the specifications
- h) modification in Q or R .

The simulation steps comprise:

- a) the solution of the Riccati equation.

It was solved through the matrix sign function [1], giving the positive definite solution.

A previous implementation using the method presented in [2] resulted more complex and dependant on the explicit storage of the coefficients of the $p_{i,j}$ as function of the A , B , Q and R elements, in the linear equations relating the $p_{i,j}$.

- b) the numerical computation of $x_i(t)$, for t in $[0, T]$, obtained from the solution

$$\underline{x}(t) = \underline{x}(t_0) \exp(M t)$$

where $M = A - BR^{-1}B^TP$. It was supposed $\underline{x}(t_0) = (1 \ 1 \ \dots \ 1)^T$.

A number of methods for computing the exponential is presented in [4]. The current implementation uses the scaling

and squaring method, which employs the property

$$\exp(Mt) = (\exp(Mt/m))^m$$

for computing first $\exp(Mt/m)$ by a power series and then $\exp(Mt)$. The factor m is chosen as a power of 2 for which $\exp(Mt/m)$ can be reliably computed.

c) the computation of the chosen parameters $p_{k,s}$ from the curves $x_i(t)$. Although the current implementation was restricted to the already mentioned four parameters, the system allows other parameters to be specified, provided that they are externally defined and their name and computing routine known to the rule environment.

The system was implemented in a version of OPS5 for an IBM-PC/XT like microcomputer and the external routines were implemented in C.

The performance of the simulation routines can be measured by some tests (average time):

order 2: 20 sec.

order 3: 35 sec.

order 4: 65 sec.

order 5: 110 sec.

As an example, it will be considered the following system:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -6 \end{bmatrix}; \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = I$$

This system has three poles and one zero. The constraints on the parameters are given by the intervals they must satisfy, for x_3 :

td in $[0.02, 0.1]$

tp in $[0.2, 0.3]$

If the initial Q and R are $Q = I$ and $R = 1$, $td \approx 0.4$ and $tp \approx 0.5$.

In order to decrease them, one suggestion is to increase q_{11} ; if $q_{11} = 10$, $td \approx 0.2$ and $tp \approx 0.45$. In order to decrease tp ,

one suggestion is to decrease q_{10} ; if $q_{10} = -5$, $td \approx 0.1$ and $tp \approx 0.4$. In order to decrease td , one suggestion is to decrease q_{20} ; if $q_{20} = -5$, $td \approx 0.05$ and $tp \approx 0.3$.

This example shows how a single modification can turn the parameters into satisfied. The system suggests estimated new matrix elements based on additional simulations performed for each matrix element.

As the order of a control system increases, it is more difficult for one to arrive at matrixes Q and R which satisfy the constraints. The suggestions given by this system, although not conclusive, may reduce the number of choices a designer would have to consider.

4. FUTURE EXTENSIONS

Other additional features that are under study are:

- the extension to the frequency domain, in order to perform some stability analysis and to improve the heuristics classification

- the use of a more general form for the specifications

- the addition of explanatory capabilities.

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Fig.4: System architecture

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