Generation of Subsynoptic-Scale Unstable Baroclinic Waves by Surface Friction

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ABSTRACT

Inclusion of surface drag produces an additional growth rate peak at $L_x \approx 1000$ km, even if the curvature ($d^2\bar{u}/dz^2$) of the basic zonal wind profile is not favorable and diabatic heating due to the liberation of latent heat not allowed for. The growth rate of this most unstable intermediate-scale wave increases with surface drag and decreases with internal friction. This wave has amplitude maxima near the surface and at about 1.5 km. A uniform friction at all depths does not produce a second growth rate peak, but it reduces the growth rate of the Eady-Charney mode.

1. Introduction

Friction prevents unduly rapid growths of certain unstable waves in their finite amplitude evolution. It may also play its own role in modifying the baroclinic instability spectrum. A few authors (Holopainen, 1961; Haltiner and Caverly, 1965) have examined the effect of friction on baroclinic instability in a simple quasi-geostrophic (QG) two-level model. Holopainen showed that the introduction of viscosity changes the linear stability curve and leads to an unexpected destabilization. Chandrasekhar (1961) comments that in rotating fluids the role of friction can be reversed.

In their study of the validity of finite difference approximations in solving dynamic stability problems, Árnason *et al.* (1967) suggested that the inclusion of friction in the models would serve to remove the singularities responsible for ambiguities in interpreting the results. Romea (1977) studied the effect of friction in finite amplitude baroclinic waves and concluded that, in limiting vanishing viscosity, steady unstable waves of finite amplitude were discovered for shears which were subcritical with respect to the inviscid threshold. Some meteorologists (Schneider and Lindzen, 1976; Tokioka, 1973) have parameterized the vertical momentum exchange (or cumulus friction), in addition to the liberation of latent heat by cumulus clouds. Tokioka noted a negligible effect of cumulus friction compared to the heating effect on the baroclinic instability of the intermediate-scale waves.

Yamasaki (1969), working on tropical, unstable baroclinic waves, introduced Rayleigh friction at the lower rigid boundary, simulating the surface drag. The surface drag contributed remarkably to increasing growth rates of, what he called, the ES mode. A similar effect was also felt by the E mode, but to a

lesser degree. The E and ES modes are essentially subsynoptic ($L_x < 3000$ km) waves in a conditionally unstable tropical atmosphere. Thus, it would be interesting to see the effects of surface friction, in contrast to that of internal friction, on the middle and high latitude baroclinic instability spectrum, which is the objective of the present note. For this purpose, a multilevel primitive equation model is adapted.

2. The eigenvalue problem

The primitive equation model considered is the same as that given in Satyamurty *et al.* (1982, hereafter referred to as SRM) except that the Rayleigh friction terms are included in the horizontal momentum equations. The final equation governing sinusoidal perturbations on a β -plane, in a meridionally

FIG. 1. Vertical profiles of the basic state zonal wind ($\bar{u}$) and potential temperature ($\bar{\theta}$).

FIG. 2. Instability spectra in cases 1-4: RII, region of the intermediate-scale instability; EADY, region of Eady-Charney instability. (See text for the cases.)

independent basic state without diabatic heating, is given by

$$A \frac{d^2 \tilde{w}}{dz^2} + B \frac{d\tilde{w}}{dz} + C\tilde{w} = 0, \quad (1)$$

where

$$\left. \begin{aligned} A &= D\delta(f^2 + D\delta'), & B &= f^2 \frac{d\tilde{u}}{dz} (\delta k^2 - D) \\ \delta &= (\bar{u} - c), & C &= D^2 \left(S - \delta \frac{d^2 \bar{u}}{dz^2} \right) \\ \delta' &= \left(\delta - \frac{i\alpha}{k} \right), & D &= (\beta - k^2 \delta) \end{aligned} \right\},$$

and where $\tilde{w}$ is the complex amplitude of the vertical velocity; c is the complex phase speed of the wave,

α the Rayleigh frictional coefficient, k the zonal wave-number, and $\bar{u}$ and S the zonal wind and static stability of the basic state; the rest of the symbols have their usual meaning. Applying Eq. (1) to a 21-level finite difference grid with the homogeneous boundary conditions, $\tilde{w}_1 = \tilde{w}_{21} = 0$ yields a numerical eigenvalue problem in which c is the eigenvalue and $\tilde{w}_n$, $n = 2, 3, \dots, 20$, constitute the eigenvector. The instability spectrum (IS) is obtained as in SRM. For this the frictional coefficient (α) and the basic state [$u(z)$, $S(z)$] must be specified.

For the vertical profile P1, described in SRM and shown in Fig. 1, the IS are obtained in the following six cases: 1) $\alpha = 10^{-5} \text{ s}^{-1}$ at all levels representing large uniform internal friction; 2) $\alpha = 10^{-6} \text{ s}^{-1}$ at all levels representing small uniform internal friction; 3)

FIG. 3. Vertical structure of the unstable intermediate-scale (dashed line) and Eady-Charney (solid line) waves. T represents trough lines; θ , potential temperature maximum; W , upward velocity maximum.

FIG. 4. Energy cycle. Numbers with parentheses indicate $L_x = 3700$ km; numbers without parentheses indicate $L_x = 1100$ km in case 3. $\bar{K}$, $\bar{P}$ indicate the kinetic and potential energy of the basic state; K' , P' the kinetic and potential energy of the growing wave; D is dissipation.

$\alpha = 10^{-5} \text{ s}^{-1}$ at the two bottom levels only, representing friction in the lowest 1 km layer due to surface drag; 4) $\alpha = 10^{-5} \text{ s}^{-1}$ at the two bottom levels and $\alpha = 10^{-6} \text{ s}^{-1}$ at all other levels, representing surface drag plus internal friction; 5) $\alpha = 10^{-5} \text{ s}^{-1}$ at one bottom level only, representing a shallow friction layer of 0.5 km depth; and 6) $\alpha = 10^{-5} \text{ s}^{-1}$ at the three bottom levels, representing a deep friction layer of 1.5 km. In cases 7 and 8, the profiles P2 and P1L, described in SRM, are respectively used with a friction layer of 1 km depth. In all these cases the bottom level is meant to be the first level above the ground ($z = 0$) at which Eq. (1) is applied, because at the ground level the equation is simply $\bar{w} = 0$. The value $\alpha = 10^{-5} \text{ s}^{-1}$ corresponds to a dissipative time scale of the order of one day, and $\alpha = 10^{-6} \text{ s}^{-1}$ corresponds to 10 days; that is, left to friction alone with $\alpha = 10^{-5} \text{ s}^{-1}$, the perturbation velocities die down in about a day. It may be noted that the surface friction is introduced into the model in much the same way as in Yamasaki (1969). The IS obtained in all these cases are discussed in the following section.

3. Results and discussion

The profile P1 is chosen for examining the effects of friction and surface drag because the curvature ($d^2\bar{u}/dz^2$) of P1 does not favor subsynoptic-scale instability (as found in SRM) and therefore any new region of instability may safely be attributed to the internal or surface friction, as the case may be. This profile is representative of the regions poleward of a surface frontal zone.

In cases 1 and 2 (see Fig. 2) the e -folding times of the Eady-Charney Modes (ECM) are higher than in the frictionless case (case 13 of SRM). Especially in case 1, where the internal friction is large, the e -folding times have increased threefold compared to those in the small friction case 2. These two cases do not present any other mode except the ECM with a most unstable wave (MUW) of $L_x \approx 4000$ km.

Case 3, where there is surface friction only, presents a new region of subsynoptic-scale instability (RII) besides the ECM. In case 4, where there is internal friction also, the e -folding times of all modes increase, thus showing a small overall stabilization. The MUW in the RII has a wavelength of about 1000 km and has an e -folding time of slightly more than two days. This region of instability is due to the surface friction. The vertical structures of the MUWs of RII ($L_x = 1100$ km) and ECM ($L_x = 3700$ km) obtained in case 3 are compared in Fig. 3. It is seen that the subsynoptic wave is confined to the lower troposphere, and its amplitude decreases rapidly with height above 2 km. The phase lines of this wave show a complex structure, although they have a westward tilt in general. It may be noted that this wave has a warm core in the lower middle troposphere (2.0–2.5 km) where the trough line and the maximum potential temperature line come close. The comparative energetics are shown in Fig. 4. The conversion from potential to kinetic energy of the subsynoptic-scale perturbation is 74%, more than double the rate in the case of ECM.

FIG. 5. As in Fig. 2 except for cases 3, 5 and 6.

FIG. 6. Vertical variation of geopotential in cases 3, 5 and 6.

The rate of increase of kinetic energy is higher than that of potential energy; the rate of dissipation of kinetic energy is large and therefore it seems reasonable to believe that these subsynoptic-scale disturbances have shorter life periods.

In cases 5 and 6, (see IS, Fig. 5), the depth of the friction layer varies and as a consequence the horizontal scales and the growth rates of the subsynoptic-scale MUW vary. The deeper the friction layer, the larger the horizontal scale and the growth rate. The relative variations of the amplitude of geopotential with height are shown in Fig. 6. Here again it is seen that the deeper the friction layer, the deeper the subsynoptic-scale disturbances. These results may, at first sight, suggest that the interface between the two regimes of friction acts as a kind of lid dividing the total vertical extent of the fluid into two parts of different depths, each responding individually to its own ECM. However, there are two reasons to believe the con-

trary. One is that $\tilde{w}$ (profile not shown here) of the subsynoptic wave attains a maximum (far from zero) near the interface; the other is that the regular ECM (shown in Fig. 5) does not show any reduction in its horizontal scale.

Yamasaki's (1969) results demonstrate the necessity of surface friction for the growth of tropical disturbances for which diabatic heating by cumulus convection is important. On the other hand, the results presented here show that in higher latitudes (on the poleward side of the surface frontal zone) the surface friction alone is capable of generating disturbances of horizontal scale $L_x \approx 1000$ km. These unstable waves have some of the characteristics that resemble the disturbances observed near the "Baiu" front (Matsumoto *et al.*, 1970).

The effect of surface friction on the IS of a linear profile P1L, ($d^2\tilde{u}/dz^2 = 0$), described in SRM, is examined in case 8. Its IS (see Fig. 7) shows very little change from that of case 3. This shows that the effects of friction and curvature are likely to be uncoupled as can be heuristically assumed from Eq. (1). The product of the terms α and $d^2\tilde{u}/dz^2$ appear only in the coefficient C . Both α/k and $\delta d^2\tilde{u}/dz^2$ are much smaller in magnitude than the terms δ and S , respectively, and therefore the term containing their product can safely be ignored compared to $k^2(S\delta^2 - 2iS\alpha/k - \delta^3 d^2\tilde{u}/dz^2)$. In case 7, the effects of the surface friction on a profile (P2) whose curvature is known to be favorable for subsynoptic-scale instability is demonstrated. It can be readily seen in Fig. 7 that this case also presents the two regions of instability, although with very little demarcation. As the growth rates and the horizontal scales of the modes in RII increase, the two regions of instability seem to have merged.

The Rayleigh friction with uniform vertical resolution may be a little too crude because a substantial part of the disturbance is found to be within the friction layer itself. If the friction is introduced in the form $(\partial/\partial z)[K(z)\partial u/\partial z]$, (as is aptly suggested by one

FIG. 7. As in Fig. 2 except for cases 3, 7 and 8.

of the reviewers), the final equation would be of the fourth order in $\tilde{w}$ instead of (1) which is of second order. To solve such a problem is cumbersome and beyond the scope of the present study. Moreover, the appropriate profile of $K(z)$ is also not well-known. Therefore, a more sophisticated modeling of the friction layer in a linear problem with the primitive equations is deferred for future study.

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