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Nonlinear generation of the fundamental radiation in plasmas: the influence of induced ion-acoustic and Langmuir waves

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A nonlinear emission mechanism of electromagnetic waves at the fundamental plasma frequency has been examined by Chian & Alves. This mechanism is based on the electromagnetic oscillating two-stream instability driven by two oppositely propagating Langmuir waves. The excitation of the electromagnetic oscillating two-stream instability is due to nonlinear wave-wave coupling involving Langmuir waves, low-frequency density waves and electromagnetic waves. In this paper the Chian & Alves model is improved using the generalized Zakharov equations. Attention is directed toward the influence of induced low-frequency and Langmuir waves on the properties of the electromagnetic oscillating two-stream instability. Presumably, the properties derived in the present context may be relevant to both space and laboratory plasmas.

1. Introduction

Nonlinear generation of coherent electromagnetic radiation by intense Langmuir waves in the vicinity of the fundamental plasma frequency f_p is of current interest in space and laboratory plasmas. In space plasmas the f_p radiation has been observed in the solar wind (Lin *et al.* 1986; Chian & Alves 1988), upstream of planetary bow shocks (Gurnett & Frank 1975; Chian 1990), at the heliopause (Kurth *et al.* 1984; Chian 1990) and in active experiments in space (Thidé, Kopka & Stubbe 1982; Chian 1991). In laboratory plasmas the f_p radiation has been observed in quiescent machines (Leung, Tran & Wong 1981; Cheng *et al.* 1981), tokamaks (Costley & TFR Group 1977; Gandy, Hutchinson & Yates 1985) and stellarators (Longinov, Perepelkin & Suprunenko 1976).

In a pioneer work Lashmore-Davies (1974) demonstrated that an efficient process for converting intense Langmuir waves (L) into f_p electromagnetic waves (T) can be achieved by two counterstreaming Langmuir pump waves through an electromagnetic (or inverse) oscillating two-stream instability. Recently, Chian & Alves (1988) and Chian (1991) extended the formalism of Lashmore-Davies (1974) to include both decay-type ($L \rightarrow T + S$) and fusion-type ($L + S \rightarrow T$) interactions as well as subsonic ($|\omega|^2 \ll c_s^2 k^2$) and supersonic ($|\omega|^2 \gg c_s^2 k^2$) regimes, where ω and k are the wave frequency and wavenumber of the

FIGURE 1. Geometry of wave-vector matching condition adopted by Chian & Alves (1988).

low-frequency density fluctuations (S), which reduce to ion-acoustic waves if the mode is a normal one.

In the works by Lashmore-Davies (1974), Chian & Alves (1988) and Chian (1991) the nonlinear wave-wave coupling only includes electromagnetic waves in the induced high-frequency modes, in addition to the induced low-frequency waves. Thus the process involves two wave triplets as depicted in figure 1. This model yields an instability that is up-conversion ($\omega_T > \omega_0$, where ω_0 and ω_T are the frequencies of the pump Langmuir and induced electromagnetic waves respectively) and purely growing ($\text{Re}(\omega) = 0$).

In this paper we adopt the generalized Zakharov equations (Kuznetsov 1974; Thornhill & ter Haar 1978; Akimoto 1988) to re-examine decay-type electromagnetic parametric instabilities. We investigate the influence of the induced Langmuir and low-frequency waves on the electromagnetic two-stream instability. Through the use of the generalized Zakharov equations, it can be seen that, in general, the instability excites both high-frequency electromagnetic and Langmuir waves (Akimoto 1988). Moreover, because of the symmetry of the wave-vector kinematics, two distinct low-frequency waves ($\pm \mathbf{k}$ and $\pm \mathbf{k} \mp 2\mathbf{k}_0 \equiv \pm \hat{\mathbf{k}}$, where $\pm \mathbf{k}_0$ are the wavenumbers of the Langmuir pumps) are excited. We show that in the presence of these induced Langmuir and low-frequency waves the behaviour of the instability is greatly modified.

The paper is organized as follows. In §2 the fundamental equations are discussed. In §3 the dispersion relation for the instability is derived. In §4 the dispersion relation is analysed both analytically and numerically. In §5 the conclusion is given.

2. Fundamental equations

The coupled wave equations (Kuznetsov 1974; Thornhill & ter Haar 1978; Akimoto 1988) that describe the coherent generation of electromagnetic and electrostatic waves by intense Langmuir waves via ponderomotive coupling with low-frequency modes are given by the generalized Zakharov equations:

$$[\partial_t^2 + \omega_p^2 + c^2 \nabla \times \nabla \times - 3v_{th}^2 \nabla(\nabla \cdot)] \mathbf{E} = -\frac{\omega_p^2}{n_0} n \mathbf{E}, \quad (1)$$

$$(\partial_t^2 - c_s^2 \Delta) n = \frac{e_0}{2m_i} \Delta \langle \mathbf{E}^2 \rangle, \quad (2)$$

where $\omega_p = (n_0 e^2 / m_e \epsilon_0)^{1/2}$, c is the velocity of light, $v_{th} = (k_B T_e / m_e)^{1/2}$, $c_s = (k_B T_e / m_i)^{1/2}$ and the angle brackets denote the fast time average.

Assuming that the electric field of the Langmuir pump wave is given in the form $\mathbf{E}_0(\mathbf{r}, t) \equiv \mathbf{E}_0(\mathbf{r}) e^{-i\omega_0 t} + \text{c.c.}$ and that the whole fast dynamics has a frequency close to the pump frequency, it is convenient to write the electric field of the induced high-frequency modes as

$$\delta \mathbf{E}_{hf} \equiv \delta \mathbf{E} e^{-i\omega_0 t} + \text{c.c.}, \quad (3)$$

with $\omega_0 |\delta \mathbf{E}| \gg \partial_t |\delta \mathbf{E}|$,

$$\omega_0 = (\omega_p^2 + 3v_{th}^2 k_0^2)^{\frac{1}{2}}, \quad (4)$$

and k_0^2 as the square of the pump wave vector. Applying the perturbation analysis to (1) and (2), we obtain

$$[-2i\omega_0 \partial_t - \delta_\omega + c^2 \nabla \times \nabla \times - 3v_{th}^2 \nabla(\nabla \cdot)] \delta \mathbf{E} = -\frac{\omega_p^2}{n_0} n \mathbf{E}_0, \quad (5)$$

$$(\partial_t^2 - c_s^2 \Delta) n = \frac{\epsilon_0}{2m_i} \Delta (\mathbf{E}_0 \cdot \delta \mathbf{E}^* + \mathbf{E}_0^* \cdot \delta \mathbf{E}), \quad (6)$$

with $\delta_\omega \equiv \omega_0^2 - \omega_p^2$.

If we now introduce the Fourier transform of a function $f(\mathbf{r})$ as

$$f(\mathbf{r}) = \int f(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k},$$

it is possible to obtain the set

$$[-2i\omega_p \partial_t - \delta_\omega - c^2 \mathbf{k} \times \mathbf{k} \times + 3v_{th}^2 \mathbf{k}(\mathbf{k} \cdot)] \delta \mathbf{E}(\mathbf{k}) = -\frac{\omega_p^2}{n_0} \int n(\mathbf{k}_1) \mathbf{E}_0(\mathbf{k} - \mathbf{k}_1) d\mathbf{k}_1 \quad (7)$$

$$d_S n = \frac{-k^2 \epsilon_0}{2m_i} \int [\mathbf{E}_0(\mathbf{k} - \mathbf{k}_1) \cdot \delta \mathbf{E}(-\mathbf{k}_1)^* + \mathbf{E}_0(\mathbf{k}_1 - \mathbf{k}) \cdot \delta \mathbf{E}(\mathbf{k}_1)] d\mathbf{k}_1 \quad (8)$$

with $d_S \equiv \partial_t^2 + \omega_S^2$, $\omega_S^2 \equiv c_s^2 k^2$ and $n(\mathbf{k}) = n(-\mathbf{k})^*$, which is a simplified version of our original set of equations.

3. Derivation of the dispersion relation

We should like to establish some connection between the induced high- and low-frequency waves generated in our system. If we could express the field $\delta \mathbf{E}$ in terms of the fluctuating density n then it should be possible to use (8) to derive a closed equation for this last variable and eventually a dispersion relation. In order to make this connection, it is apparent that we must employ (7). This equation, being of quite a general form, will enable us to focus the attention on a similar model (with the same pump spectrum) to that analysed in Chian & Alves (1988) (hereinafter referred to as paper I). Following paper I, we consider two Langmuir pumps $\mathbf{E}_0(\pm \mathbf{k}_0)$ that propagate oppositely along a longitudinal (x) axis, generating two opposite electromagnetic induced modes emitted primarily along transverse (y) axis, plus, as we shall see, a number of previously not considered induced Langmuir modes that propagate primarily along the x axis (Akimoto 1988).

In this paper we impose perfect $\mathbf{k}$ -matching, but introduce frequency mismatches between the interacting waves.

FIGURE 2. Generation of the induced electromagnetic modes with wave vectors k_T (a) and $-k_T$ (b).

FIGURE 3. Generation of the induced Langmuir modes $\pm k_L$ and $\pm k̂_L$.

The kinematic conditions for all the induced modes coupled to the pumps via low-frequency oscillations are shown in figures 2 and 3. In figure 2(a) (respectively 2b) it is seen that the decay of the pumps $\pm k_0$ generate low-frequency modes $-k$ (respectively $k̂$) and $-k̂$ (respectively k), and a transverse mode propagating with wavevector k_T (respectively $-k_T$). Using the same logic, in figure 3 we see that a number of Langmuir-induced modes are also generated during the process, it being possible to classify these modes as electrostatic only because we have used the assumption of very small $|k_y|$. We shall see that these modes, although not considered before, are of great relevance for the dynamics of the instability.

The coupling between all the modes is provided by low-frequency fluctuations whose wave vectors we take as subject to the conditions $|k_x|_{\text{low-frequency}} = |k_0|$ and $|k_y|_{\text{low-frequency}} \ll |k_0|$ since it is only in this kind of geometry that the electromagnetic modes are not off-resonant (see figure 2, recalling that the coefficient of k^2 for electromagnetic waves is much larger than that for electrostatic waves). In paper I only one of these low-frequency modes was employed to describe the process. In the present paper, however, we shall work

with two of these modes (besides the complex conjugates), because only the simultaneous presence of two such modes are compatible with the symmetry of our system. This symmetry can be appreciated either with help of figure 2 or, in a more formal way, with help of the calculations that follow later on.

As a consequence of this symmetry, two symmetrically located low-frequency modes are present. This appears to be a new result; we shall show that, depending essentially on the relative phases of these two modes, the electromagnetic waves can either vanish or be reinforced (in relation to the results of paper I), because, as one can see from figure 2(a) for example, the production of each transverse wave depends on both low-frequency waves.

Next, we proceed with our analytical calculations in order to check the validity of the (geometrical) comments made above. To do that, we begin by writing the total high-frequency fluctuating field in terms of its transverse and longitudinal components as follows:

$$\delta \mathbf{E} = \delta \mathbf{E}_L + \delta \mathbf{E}_T, \quad (9)$$

where $\delta \mathbf{E}_{L,T} = \delta E_{L,T} \hat{\mathbf{x}}$.

With this in perspective, we can use (7) to derive the following relations (Smith, Goldstein & Papadopoulos 1979):

$$\begin{aligned} \delta E_L(\mathbf{k}) &= \frac{\omega_p^2}{2i\omega_p \partial_t + \delta_\omega - 3v_{th}^2 k^2} \int \frac{n(\mathbf{k}_1)}{n_0} E_0(\mathbf{k} - \mathbf{k}_1) d\mathbf{k}_1, \\ \delta E_T(\mathbf{k}) &= \frac{\omega_p^2}{2i\omega_p \partial_t + \delta_\omega - c^2 k^2} \int \frac{n(\mathbf{k}_1)}{n_0} E_0(\mathbf{k} - \mathbf{k}_1) d\mathbf{k}_1. \end{aligned} \quad (10)$$

The next step is to substitute (10) into (8) to obtain the sought-for closed equation for the low-frequency fluctuations. Doing this, we arrive at the relation

$$\begin{aligned} d_S n &= \frac{-k^2 \epsilon_0 \omega_p^2}{2m_i n_0} \iint \left[E_0(\mathbf{k} - \mathbf{k}_1) n(\mathbf{k}_2)^* E_0(-\mathbf{k}_1 - \mathbf{k}_2) \right. \\ &\times \left(\frac{1}{-2i\omega_p \partial_t + \delta_\omega - c^2 k_1^2} + \frac{1}{-2i\omega_p \partial_t + \delta_\omega - 3v_{th}^2 k_1^2} \right) + E_0(\mathbf{k}_1 - \mathbf{k}) n(\mathbf{k}_2) E_0(\mathbf{k}_1 - \mathbf{k}_2) \\ &\times \left. \left(\frac{1}{2i\omega_p \partial_t + \delta_\omega - c^2 k_1^2} + \frac{1}{2i\omega_p \partial_t + \delta_\omega - 3v_{th}^2 k_1^2} \right) \right] d\mathbf{k}_1 d\mathbf{k}_2. \end{aligned} \quad (11)$$

An alternative way to approach the problem would be to use all the kinematic information present in figures 2 and 3 in order to calculate the integrals of (10) immediately. This approach, although somewhat less formal than the one we are presently using, seems to be physically more appealing, because the mode generation emerges much more clearly, as can be seen in appendix A.

Now, we make use of the pump-wave spectrum in order to evaluate the integrals present in (11). To do so, the only thing we have to remember again is that, according to the present model, we have two Langmuir pump waves propagating oppositely along the x axis. This fact, along with the assumption that both modes have the same amplitude and phase, enables us to write

$$E_0(\mathbf{k}) = \delta(k_y) [\delta(k_x - k_0) - \delta(k_x + k_0)] E_0. \quad (12)$$

Finally, by considering again the fact that in order to have a resonant process, the wave vectors of the electromagnetic modes must be much smaller than those relative to the Langmuir waves, we obtain the following relation for the $\mathbf{k}$ component of the low-frequency fluctuations (see appendix B for a more detailed calculation):

$$d_S n(\mathbf{k}) = (\Lambda_{1-} + \Lambda_{1+}) [n(\mathbf{k}) - n(-2\mathbf{k}_0 + \mathbf{k})] + (\Lambda_{2-} + \Lambda_{2+}) [n(\mathbf{k}) - n(2\mathbf{k}_0 + \mathbf{k})] \quad (13)$$

$$\text{where} \quad \Lambda_{1\pm} \equiv \frac{-k^2 \epsilon_0 \omega_p^2 E_0^2}{2m_i n_0} \frac{1}{\pm 2i\omega_p \partial_t + \delta_\omega - c^2(\mathbf{k} - \mathbf{k}_0)^2}, \quad (14)$$

$$\Lambda_{2\pm} \equiv \frac{-k^2 \epsilon_0 \omega_p^2 E_0^2}{2m_i n_0} \frac{1}{\pm 2i\omega_p \partial_t + \delta_\omega - 3v_{th}^2(\mathbf{k} + \mathbf{k}_0)^2}. \quad (15)$$

From these equations, we see, as mentioned before, that we did obtain an equation for the low-frequency field solely. The problem is that, owing to the nonlinearities present in the model, we could not get a closed form involving just one of the Fourier components. Indeed, from (13) it is possible to see that the component $\mathbf{k}$ is coupled to the components $-2\mathbf{k}_0 + \mathbf{k}$ and $2\mathbf{k}_0 + \mathbf{k}$, and it is not hard to verify that such a coupling eventually interconnects all the components of the type $2p\mathbf{k}_0 + \mathbf{k}$, with p an integer. With our aim the truncation of the chain of equations thus obtained, we shall consider only those low-frequency modes defined by the wave-vectors $\mp 2\mathbf{k}_0 \pm \mathbf{k}$ and $\mathbf{k}$. This appears to be a reasonable approximation (as will be shown in §4.3 and appendix C), because, as can be seen with the help of an extrapolation of figure 3, modes with higher values of the wave vector are related to the generation of induced Langmuir oscillations that themselves have larger wave vectors, therefore being much less resonant than those pertaining to the original resonant multiplet represented in the figures.

Now if we use our approximate truncation condition $n(2\mathbf{k}_0 + \mathbf{k}) \approx 0$, we can write the complete set of equations for the low-frequency components $\mathbf{k}$ and $-2\mathbf{k}_0 + \mathbf{k}$ in the form

$$d_S n(\mathbf{k}) = \Lambda_1 [n(\mathbf{k}) - n(-2\mathbf{k}_0 + \mathbf{k})] + \Lambda_2 n(\mathbf{k}), \quad (16)$$

$$d_S n(-2\mathbf{k}_0 + \mathbf{k}) = \Lambda_1 [n(-2\mathbf{k}_0 + \mathbf{k}) - n(\mathbf{k})] + \Lambda_2 n(-2\mathbf{k}_0 + \mathbf{k}), \quad (17)$$

where $\Lambda_i \equiv \Lambda_{i-} + \Lambda_{i+}$.

These equations show an interesting symmetry underlying the wave dynamics. There are two eigenmodes, $n_+ = n(\mathbf{k}) + n(-2\mathbf{k}_0 + \mathbf{k})$ and $n_- = n(\mathbf{k}) - n(-2\mathbf{k}_0 + \mathbf{k})$, governed by the equations

$$d_S n_+ = \Lambda_2 n_+, \quad (18)$$

$$d_S n_- = (2\Lambda_1 + \Lambda_2) n_- \quad (19)$$

respectively.

In the first case no electromagnetic radiation is observed because of the absence of the resonant denominator responsible for this kind of process contained in the factor Λ_1 ; here only induced Langmuir waves with wave vectors $(\mathbf{k} + \mathbf{k}_0)^2$ are produced.

In the second case both electrostatic and electromagnetic radiation are present because both of the factors Λ_1 and Λ_2 appear. In physical terms, what is happening is that, while in the first case the low-frequency modes interfere

destructively, with no transverse modes being produced, in the second the interference is constructive, leading to the generation of these modes. In the present work we are mostly interested in the generation of transverse modes; therefore from now on we shall focus our analysis on this situation.

If, then, we take our governing equation (19) and make the harmonic substitution $\partial_t \rightarrow -i\omega$, we can finally obtain a dispersion relation for our parametric process in the form

$$\omega^2 - \omega_S^2 = -\frac{1}{8}\omega_p \omega_S^2 W \left(\frac{2\delta_T}{\delta_T^2 - \omega^2} + \frac{\delta_L}{\delta_L^2 - \omega^2} \right), \quad (20)$$

where

$$\delta_T \equiv \frac{c^2(\mathbf{k} - \mathbf{k}_0)^2 - \delta_\omega}{2\omega_p} \approx \omega_T - \omega_0,$$

$$\delta_L \equiv \frac{3v_{th}^2(\mathbf{k} + \mathbf{k}_0)^2 - \delta_\omega}{2\omega_p} \approx 9v_{th}^2 k_0^2 \approx \omega_L - \omega_0$$

and $W \equiv \epsilon_0 E_0^2 / n_0 k_B T_e$ is the normalized energy of the pump waves. In deriving the dispersion relation (20), the following waves have been taken into consideration in the interaction, as shown in figures 2 and 3: two counter-streaming pump Langmuir waves $\mathbf{E}_0^+(\omega_0, \mathbf{k}_0)$ and $\mathbf{E}_0^-(\omega_0, -\mathbf{k}_0)$; two distinct low-frequency density waves $n(\omega, \pm \mathbf{k})$ and $n(\omega, \pm \hat{\mathbf{k}})$; four induced Langmuir waves $\delta \mathbf{E}_L(\omega_0 + \omega, \mathbf{k}_0 + \mathbf{k} \equiv \mathbf{k}_L)$, $\delta \mathbf{E}_L(\omega_0 + \omega, -\mathbf{k}_0 + \hat{\mathbf{k}} \equiv \hat{\mathbf{k}}_L)$, $\delta \mathbf{E}_L(\omega_0 - \omega, -\mathbf{k}_0 - \mathbf{k} \equiv -\mathbf{k}_L)$ and $\delta \mathbf{E}_L(\omega_0 - \omega, \mathbf{k}_0 - \hat{\mathbf{k}} \equiv -\hat{\mathbf{k}}_L)$; and two induced electromagnetic waves $\delta \mathbf{E}_T(\omega_0 + \omega, -\mathbf{k}_0 + \mathbf{k} = \mathbf{k}_0 + \hat{\mathbf{k}} \equiv \mathbf{k}_T)$ and $\delta \mathbf{E}_T(\omega_0 - \omega, \mathbf{k}_0 - \mathbf{k} = -\mathbf{k}_0 - \hat{\mathbf{k}} \equiv -\mathbf{k}_T)$.

Note that, apart from a factor of order unity, the parameter δ_T is the electromagnetic detuning factor δ introduced in paper I; δ_T can be either greater or less than zero. The parameter δ_L is an electrostatic detuning factor that arises here owing to the presence of the induced Langmuir waves; δ_L is always greater than zero for the wave-vector configuration considered in this paper.

The dispersion relation we have just obtained describes the stability properties of the wave multiplet represented in figures 2 and 3, in §4 we shall obtain some of its analytical and numerical characteristics.

4. Analysis of the dispersion relation

To begin with, it is interesting to compare the dispersion relation (20) with the similar relation obtained in paper I. In this regard we emphasize that two new circumstances have been considered here: we have taken into account both the presence of the second low-frequency mode and the induced Langmuir waves. We have already analysed one of the roles of this second low-frequency mode – our conclusion is that its presence may cause the electromagnetic radiation to vanish (the destructive-interference case) or be reinforced, with a growth rate greater to the one previously calculated (the constructive-interference case). As for the induced Langmuir modes, we may say that the basic effect, as we shall see, is to enlarge the regions of instability for the system.

Now, let us study various regimes of our model.

4.1. Resonant cases ($\omega^2 \approx \omega_S^2$)

This is the case where the frequency is very close to ω_S : $\omega = \omega_S + i\Gamma$, with $|\Gamma| \ll |\omega_S|$. We have then instabilities with the pumps decaying into high-frequency and normal ion-acoustic waves. There are two solutions for (20): (i) $\delta_T^2 \approx \omega_S^2 \mp \delta_L^2$; (ii) $\delta_T^2 \approx \delta_L^2 \approx \omega_S^2$.

In the first case we have essentially a parametric decay instability in which the pumps decay into transverse radiation and ion-acoustic waves. Then we must have (Lashmore-Davies 1974; Shukla *et al.* 1983; Lin *et al.* 1986; Chian & Alves 1988)

$$\delta_T \approx -\omega_S, \quad (21)$$

which requires

$$k_0 > \frac{2 m_e \omega_p}{3 m_i c_S}. \quad (22)$$

The growth rate is

$$\Gamma = \frac{1}{4}(-\delta_T \omega_p W)^{\frac{1}{2}} \approx \frac{1}{4}(\omega_S \omega_p W)^{\frac{1}{2}}, \quad (23)$$

from which we see that the instability is present whenever $W\omega_p/\omega_S \ll 1$.

In the second case we have essentially a stimulated modulational instability (Hasegawa 1970; Mima & Nishikawa 1984) in which induced electromagnetic and Langmuir waves are excited simultaneously. In order for instability to occur, here we must have

$$\delta_T \approx -\delta_L \approx -\omega_S. \quad (24)$$

The growth rate is

$$\Gamma = (\frac{1}{32}\omega_p \omega_S W)^{\frac{1}{2}}. \quad (25)$$

Another type of resonant decay would be the one involving induced Langmuir waves in lieu of transverse waves. However, since the frequency of the pump waves is smaller than that of these induced modes ($\delta_L > 0$), the decay does not take place in this situation.

4.2. Non-resonant cases ($\omega^2 \neq \omega_S^2$)

The non-resonant cases are those where ω is not close to ω_S . Considering the purely growing case $\omega = i\Gamma$, we can rewrite our dispersion relation (20) in the form

$$\Gamma^2 + \omega_S^2 = \frac{1}{8}\omega_p \omega_S^2 W \left(\frac{2\delta_T}{\delta_T^2 + \Gamma^2} + \frac{\delta_L}{\delta_L^2 + \Gamma^2} \right). \quad (26)$$

From this we see that at least one root is present when

$$\omega_S^2 < \frac{1}{8}\omega_p \omega_S^2 W \left(\frac{2}{\delta_T} + \frac{1}{\delta_L} \right), \quad (27)$$

which, as usual, establishes a threshold for the value of W as well (for comparison see Rizzato 1990) because, as Γ^2 increases from zero, the left-hand side of (26) increases from a value that is below the maximum of the right-hand side. If there are no induced Langmuir waves then from (27) we arrive at the usual conclusion (see paper I) that for the electromagnetic oscillating two-stream instability, δ_T must be a positive number, which means that the electromagnetic waves are upshifted in frequency relative to the pumps. Now,

FIGURE 4. Plots of both the right- and left-hand sides of (26) versus Γ^2 : (a) $\delta_T > 0$, implying one root; (b) $\delta_T < 0$, implying two roots.

if induced Langmuir modes are present, we see that the condition $\delta_T > 0$ can be relaxed; it suffices to have a (non-zero) sufficiently small value for δ_L . In other words, the influence of the new electrostatic modes can cause the frequency of electromagnetic waves to be downshifted in relation to the pumps. In general, it is seen that roots for $|\delta_T| \gg |\delta_L|$ with $\delta_L \neq 0$ are possible if $\omega_S^2 < \frac{1}{8}\omega_p \omega_S^2 W/\delta_L$, namely,

$$W > 8 \frac{\delta_L}{\omega_p} \quad (\delta_L \neq 0); \quad (28)$$

in the case of no induced Langmuir modes, such roots were absent. It is still interesting to note that if $-\delta_T \ll \delta_L$, with the condition (28) holding, there are two roots, one much smaller than the other, because the multiple curvatures of the right-hand side of (26) enable a double intersection of the left- and right-hand side curves. Another relevant comment that can be made is that when $W \ll 8\delta_L/\omega_p$, the Langmuir modes do not have any decisive influence on the instability and may be discarded, as was done in paper I.

All these situations can be better appreciated with the help of figure 4 where we plot both the left- and right-hand sides of (26) with the respective intersections (roots). In figures 4(a) and (b) we represent the cases $\delta_T > 0$ and $\delta_T < 0$ respectively, with $\delta_T/\omega_S = \pm 0.005$, $\delta_L = 0.5$ and $\frac{1}{8}W\omega_p/\omega_S = 1$.

4.3. Numerical solutions of the dispersion relation

In figure 5 we compare the growth rates of purely growing modes using the parameters $\frac{1}{8}W = 10^{-8}$, $\omega_p/\omega_S = 10^7$ and $\delta_L/\omega_S = 0$ (a), 0.075 (b), 0.704 (c) with $\Gamma_N \equiv \Gamma/\omega_S$ and $\delta_N \equiv \delta_T/\omega_S$. In (a) and (c) the condition (28) does not hold, and there are roots only for positive values of δ_T . For (b) this condition holds, and there are roots for negative as well as positive values of δ_T ; also, as mentioned before, the instability range is enlarged.

Finally, in figure 6 we give a general view of the processes as a function of δ_N , depicting all the unstable modes, with solid and dashed lines respectively representing the imaginary and real parts of the frequencies obtained from our dispersion relation with $\delta_L/\omega_S = 0.075$ and all other parameters the same as in figure 5. What we see may be put in the following terms: for large and negative values of δ_N , three modes are present; two of them are real ($\omega \approx -\delta_T, \omega_S$) and one (b) represents a purely growing instability, because, in the numerical range

FIGURE 5. Growth rates for the purely growing case. The values of the parameters can be found in the text.

FIGURE 6. General view of the instabilities. Solid lines represent the imaginary parts of the frequencies of the unstable modes, and dashed lines the real parts. The values of the parameters as given in the text.

considered, the condition (28) is valid. As we increase the value of δ_N , the real modes merge, giving rise to one of the resonant unstable modes mentioned before (a). For even larger values of δ_N , a very complicated structure (not seen in figure 5 because of the scales used) appears. For small (and negative) values of the parameter δ_N , two purely growing modes are present; as we go to slightly smaller values of δ_N , these two modes merge into a single mode that is not purely imaginary. In figure 6 we have not detailed the range of positive and large values of δ_N . However, we may see that in that range, besides the purely growing mode (b), there will be two propagating ones. As the dispersion relation depends only on the squares of the frequencies, figure 6 is symmetric with respect to reflections in the horizontal axis.

It is interesting to point out that if we had a set of parameters for which the condition (28) were false then the essential difference from the processes represented in figure 6 would be the absence of purely growing waves for negative values of δ_N .

Use of the truncation condition $n(2\mathbf{k} + \mathbf{k}_0) \approx 0$ has considerably simplified the calculations, but its validity must be tested. To do so, we have proceeded one step further in the derivation of the dispersion relation in order to study the role

FIGURE 7. Comparison of the growth rate Γ_N as calculated from the original (a) and refined (b) dispersion relations.

of the neglected higher-order terms. The detailed evaluation of the refined higher-order dispersion relation can be found in appendix C. What can be globally concluded is that, for the typical parameters used in this work, higher-order corrections have little influence on the value of the growth rates, as can be seen with help of figure 7, where we plot Γ_N versus δ_N for $(\omega_p/\omega_S)W = 0.1$ and $\delta_L = 0.7$, as calculated from (20) (curve (a)) and (C 7) (curve (b)). Moreover, the numerical solution of the refined dispersion relation indicates that the higher the value of k_0 , the smaller the corrections.

5. Conclusions

In this paper we have studied the nonlinear generation of fundamental radiation by two counter-streaming Langmuir waves, based on the generalized Zakharov equations.

For the decay-type interactions, we have shown that, in general, in addition to the induced Stokes electromagnetic waves and low-frequency waves with $\mathbf{k}_S = \pm \mathbf{k}$, anti-Stokes Langmuir waves and low-frequency waves with $\mathbf{k}_S = \pm \mathbf{k} \mp 2\mathbf{k}_0$ are also excited. In the presence of these induced Langmuir and low-frequency waves, the instability is not always up-conversion and pure growing; depending upon the relation between the electromagnetic (δ_T) and electrostatic (δ_L) detuning factors, down-conversion as well as instabilities with a real component of the frequency are also possible. As a new and interesting particular case, we have shown that when $\delta_T = -\delta_L$ (< 0), a new stimulated modulational instability is present in which both the induced electromagnetic and Langmuir waves are generated simultaneously.

Dissipative damping of the induced waves introduces a threshold amplitude for the onset of the parametric instability. The effect of damping can be included by replacing ∂_t^2 by $\partial_t^2 + \Gamma_i \partial_t$ in the generalized Zakharov equations (1) and (2), where Γ_i are the damping rates of the induced waves. The resulting dispersion relation becomes a function of the damping rates. For example, in the resonant case (ii) in §4.1 with $\omega^2 \approx \omega_S^2 \approx \delta_T^2 \approx \delta_L^2$ and $\Gamma_T \approx \Gamma_L$, the threshold condition is $W \geq \Gamma_S \Gamma_T / \omega_S \omega_p$. In the absence of induced Langmuir waves, $W \ll 8\delta_L/\omega_p$, the threshold conditions for both resonant and non-resonant cases reduce to the results of paper I and Lashmore-Davies (1974).

To summarize, it can be seen that processes occurring in the presence of induced Langmuir oscillations are much more complex than in their absence. It is seen that the basic role of Langmuir waves is to enlarge the range of instability, even creating some new unstable modes.

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Appendix A. Alternative derivation of (16) and (17)

Here we give an alternative way of deriving the dispersion relation.

Let us begin by using the spectral form of the low-frequency fluctuations and pump waves in (10). Then, denoting $n(\pm \mathbf{k}) \equiv n_1^\pm$, $n(\mp 2\mathbf{k}_0 \pm \mathbf{k}) \equiv n_2^\pm$ and $E_0(\pm \mathbf{k}_0) \equiv E_0^\pm$, we see from figures 2 and 3 that the only non-trivial Fourier components of the fields δE_L and δE_T are given respectively by

$$\left. \begin{aligned} \delta E_L(\mathbf{k}_L) &\equiv E_{L,1}^+ = D_L n_1^+ E_0^+, \\ \delta E_L(\hat{\mathbf{k}}_L) &\equiv E_{L,2}^+ = D_L n_2^+ E_0^+, \\ \delta E_L(-\mathbf{k}_L) &\equiv E_{L,1}^- = D_L n_1^- E_0^-, \\ \delta E_L(-\hat{\mathbf{k}}_L) &\equiv E_{L,2}^- = D_L n_2^- E_0^- \end{aligned} \right\} \quad (\text{A } 1)$$

$$\left. \begin{aligned} \delta E_T(\mathbf{k}_T) &\equiv E_T^+ = D_T(n_1^+ E_0^- + n_2^+ E_0^+), \\ \delta E_T(-\mathbf{k}_T) &\equiv E_T^- = D_T(n_1^- E_0^+ + n_2^- E_0^-). \end{aligned} \right\} \quad (\text{A } 2)$$

$$\text{where} \quad D_L \equiv \frac{\omega_p^2}{2i\omega_p \partial_t + \delta_\omega - 3v_{th}^2 k^2}, \quad D_T \equiv \frac{\omega_p^2}{2i\omega_p \partial_t + \delta_\omega - c^2 k^2},$$

from which we see that the two low-frequency modes in association with the pumps do indeed generate four Langmuir and two electromagnetic induced waves.

Now let us use the matching conditions of figures 2 and 3 along with (8) and (A 1). We have

$$d_S n_1^+ = \frac{-k^2 \epsilon_0}{2m_i} (E_0^+ E_T^{*-} + E_0^- E_{L,1}^{*-} + E_0^{*+} E_{L,1}^+ + E_0^{*-} E_T^+), \quad (\text{A } 3)$$

$$d_S n_2^+ = \frac{-k^2 \epsilon_0}{2m_i} (E_0^- E_T^{*-} + E_0^+ E_{L,2}^{*-} + E_0^{*-} E_{L,2}^+ + E_0^{*+} E_T^+). \quad (\text{A } 4)$$

If we now substitute (A 1) and (A 2) into (A 3) and (A 4), considering pumps with $E_0^- = -E_0^+ \equiv -E_0$ and remembering that $n^{*-} = n^+$, we obtain the following pair of governing equations for the low-frequency fluctuations:

$$d_S n_1^+ = \frac{-k^2 E_0^2 \epsilon_0}{2m_i} (D_L + D_L^*) n_1^+ + (D_T + D_T^*) (n_1^+ - n_2^+), \quad (\text{A } 5)$$

$$d_S n_2^+ = \frac{-k^2 E_0^2 \epsilon_0}{2m_i} (D_L + D_L^*) n_2^+ + (D_T + D_T^*) (n_2^+ - n_1^+), \quad (\text{A } 6)$$

which are nothing but (16) and (17).

Appendix B. Derivation of (13)

Let us sketch the derivation of the relation (13). To do this, we begin by inserting the spectral form (12) into (11). Considering only the $n(\mathbf{k}_2)^*$ term, we see that the non-vanishing contributions are those where $\mathbf{k} - \mathbf{k}_1 = \pm \mathbf{k}_0$ and $-\mathbf{k}_1 - \mathbf{k}_2 = \pm \mathbf{k}_0$. Let us first consider the case $\mathbf{k} - \mathbf{k}_1 = +\mathbf{k}_0$. Then we have $\mathbf{k}_1 = \mathbf{k} - \mathbf{k}_0$, which means that the only denominator we have to consider is the one responsible for generation of transverse waves; because of the smallness of k_1^2 , the one connected with induced Langmuir modes is vanishingly small.

With these considerations, and taking into account that in the present case the allowed values for $\mathbf{k}_2$ are $2\mathbf{k}_0 - \mathbf{k}$ and $-\mathbf{k}$, we obtain

$$n_{\text{term}}^* = \Lambda_{1-}(n(\mathbf{k}) - n(-2\mathbf{k}_0 + \mathbf{k})). \quad (\text{B } 1)$$

The other terms are obtained in a similar fashion.

Appendix C. Testing the validity of the truncation procedure

If one goes one step further in the derivation of the dispersion relation, additional higher-order modes are needed; one has to include in the formalism new low-frequency fluctuations of the type $n(\pm 2\mathbf{k}_0 \pm \mathbf{k})$ and $n(\mp 4\mathbf{k}_0 \pm \mathbf{k})$, and the correlated new induced Langmuir modes with wave vectors given by $\pm 3\mathbf{k}_0 \pm \mathbf{k}$ and $\pm 5\mathbf{k}_0 \mp \mathbf{k}$. In that case the truncation procedure is redefined as $n(\pm 4\mathbf{k}_0 \pm \mathbf{k}) \approx 0$, and the use of (11) in the way sketched in appendix B leads to the derivation of an augmented coupled set:

$$d_S n_1 = \Lambda_1(n_1 - n_2) + \Lambda_2(n_1 - n_3), \quad (\text{C } 1)$$

$$d_S n_2 = \Lambda_1(n_2 - n_1) + \Lambda_2(n_2 - n_4), \quad (\text{C } 2)$$

$$d'_S n_3 = \frac{k'^2}{k^2} \Lambda_2(n_3 - n_1) + \Lambda_3 n_3, \quad (\text{C } 3)$$

$$d'_S n_4 = \frac{k'^2}{k^2} \Lambda_2(n_4 - n_2) + \Lambda_3 n_4, \quad (\text{C } 4)$$

where we abbreviate $n_{1,2}^+$ by $n_{1,2}$ and introduce $n(2\mathbf{k}_0 + \mathbf{k}) \equiv n_3$, $n(-4\mathbf{k}_0 + \mathbf{k}) \equiv n_4$,

$$\Lambda_3 \equiv \frac{k^2 c_0 \omega_p^2 E_0^2}{2m_i n_0} \frac{\delta_{L'}}{\omega^2 - \delta_{L'}^2},$$

$$\delta_{L'} \equiv 3v_{\text{th}}^2(2\mathbf{k}_0 + \mathbf{k})^2 - \delta_\omega \approx 4\delta_L, \quad d'_S \equiv \partial_t^2 + c_S^2 k'^2 \equiv \partial_t^2 + c_S^2(2\mathbf{k}_0 + \mathbf{k})^2.$$

Equations (C 1)–(C 4) can be reduced to a coupled pair for the functions n_- and $n_{3-} \equiv n_3 - n_4$:

$$d_S n_- = (2\Lambda_1 + \Lambda_2) n_- - \Lambda_2 n_{3-}, \quad (\text{C } 5)$$

$$d'_S n_{3-} = \frac{k'^2}{k^2} [\Lambda_2(n_{3-} - n_-) + \Lambda_3 n_{3-}], \quad (\text{C } 6)$$

from which one can determine the corresponding augmented dispersion relation

$$\left[d'_S - \frac{k'^2}{k^2} (\Lambda_2 + \Lambda_3) \right] [d_S - (2\Lambda_1 + \Lambda_2)] = \frac{k'^2}{k^2} \Lambda_2^2. \quad (\text{C } 7)$$

In order to illustrate the role of the high-order terms included above, we have solved (C 7) numerically for

$$\frac{\omega_p}{\omega_s} \frac{W}{8} = 0.1, \quad \frac{\delta_L}{\omega_s} = 0.7,$$

which produces a graphical representation as depicted in figure 7 in the main text.

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