Two-Layer Model of Summer Circulation on the Southeast U.S. Continental Shelf

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(Manuscript received 28 August 1986, in final form 23 October 1987)

ABSTRACT

The summer circulation in the South Atlantic Bight is investigated using a two-layer finite element model. Simulations using a steady state mean summer wind field lead to the following conclusions: (i) the adjustment time of the shelf circulation to sudden changes in the wind varies between 12 to 24 hours; (ii) the main dynamical balances were geostrophic in the cross-shelf direction and with the Coriolis, pressure gradient, wind and bottom stresses all being significant in the alongshore momentum balance. Both findings agree with data analysis of Lee and Pietrafesa. This experiment produced an upwelling cell that matched well with the summer upwelling observed north of Cape Canaveral. Results of a second experiment using a 9-day real wind event compared reasonably well against observed coastal sea level, midshelf currents and an upwelling-downwelling event. Momentum balance results, associated with the short adjustment time compared to the time scale of the wind forcing (4 to 12 days), indicate that quasi-steady state conditions are dominant with a dynamical balance similar to the arrested topographic wave model of Csanady. A third experiment where the model was forced with an alongshelf pressure gradient applied along the open offshore boundary, characteristic of the Gulf Stream, also produced an upwelling cell north of Cape Canaveral. These findings support the idea that Cape Canaveral upwelling is determined by the joint effect of wind and Gulf Stream intrusions over the shelf.

1. Introduction

The southeast U.S. continental shelf, sometimes referred to as the South Atlantic Bight (SAB), extends from Cape Canaveral, Florida, to Cape Hatteras, North Carolina (Fig. 1). It is a wide and shallow continental shelf, with a width varying from 50 km off Cape Canaveral, to a maximum of 120 km off Savannah, Georgia, and reaching a minimum off Cape Hatteras. The bottom topography of this region is relatively smooth, with isobaths tending to follow the coastline. This regular pattern is, however, modified by shoals present at Cape Canaveral, Cape Fear, Cape Lookout, and Cape Hatteras. The typical depth at the shelf break is 75 m. A prominent feature in the region is the presence of the strong northward flowing Gulf Stream with its western edge located at ± 15 km from the shelf break in the region south of 32°N (Bane and Brooks, 1979).

During the summer of 1981 an interdisciplinary study of the physical, chemical and biological processes in the SAB shelf waters was undertaken by several southeast research institutions (Blanton et al., 1984). This study was termed GABEX-II, for Georgia Bight

Experiment/summer 1981, and involved hydrographic and current meter data collection, theoretical studies and numerical modeling. The study to be described here was developed as part of this research effort, and has as its primary goal the implementation and test of a numerical model suitable to describe the summer circulation in the SAB. The model is used to improve the understanding of the dynamics of this region and as a tool to predict the transport of waterborne materials.

A two-layer model is used because hydrographic data collected in the region during the summer months show that a midwater thermocline (pycnocline) is well-developed and separates the water column into two distinct layers (see Fig. 2). This is in contrast to the winter months when the shelf is vertically well mixed (Atkinson et al., 1983).

A relatively small number of coastal circulation models have been developed with the specific purpose of studying the SAB circulation. Kantha et al. (1981) report on the application of an extended version of a diagnostic model developed by Kantha (1980). This is a steady state vorticity equation model solved by integration along the characteristics (f/H) of the region to determine the transport and elevation fields. The density field, together with a second moment turbulence closure model, is then used to determine the ver-

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FIG. 1. Map of the SAB with positions where data were collected for the GABEX-II experiment (from 1 June to 15 October 1981).

tical structure of the currents. The results are in the form of climatological currents at various depths and for various seasons. The shelf surface velocities produced by the climatological wind stresses were determined to be generally 5 to 20 cm s^{-1} . Later improved versions of this model have been used also to study SAB circulation (Kantha et al., 1982; Blumberg and Mellor, 1983). Lee et al. (1982) used a diagnostic model developed by Galt (1975) to study the summer circulation on the Georgia shelf. This is also a steady state model which uses the finite element method to solve a depth integrated vorticity equation for the sea surface elevation. The horizontal transports are then determined from the sea surface elevation field (barotropic component), and the horizontal mass field (the baro-

clinic component) using the geostrophic relation. Their results indicated that the model could be useful for predicting mean flow conditions in the midshelf region (20–40 m depth), leading, however, to unsatisfactory results for the outer shelf due to the presence of the Gulf Stream, and for the inner shelf due to the shallow depths. Kourafalou et al. (1984) applied a one-layer vertically integrated finite element model to study the winter dynamics and circulation of the SAB. Model results for astronomical tide and wind forced flows showed good agreement against observed flows for the midshelf region, while poor comparisons were obtained for the outer shelf due to Gulf Stream influence on measurements. Model performance in the inner shelf could not be determined because of lack of data. The

FIG. 2. Typical vertical cross-sections of temperature, salinity and sigma- t for the SAB collected during the summer of 1981.

present study intends to extend the results of these previous efforts by including the analysis of the time dependent aspects of the circulation, and by extending the time dependent results of Kourafalou et al. (1984) to the stratified case.

The data used in this investigation are from the GABEX-II Experiment and consist of coastal wind and sea level observations, buoy winds, moored current meter time series and hydrographic measurements. Detailed analyses of these data can be found in Lee and Pietrafesa (1988) and Atkinson et al. (1988). Time series data have been filtered to remove tidal and inertial fluctuations, but no attempt has been made to remove Gulf Stream effects, which occur in the same 2–12 day period band as synoptic weather events.

2. Model description

A simple model capable of representing the vertically stratified water column during summer was chosen. This model derives from a finite element formulation by Wang and Connor (1975) and consists of two

stacked layers with constant but different densities and is known as a two layer model. Several simplifications are introduced, nonlinear advective terms are ignored because of the small velocities ($<30 \text{ cm s}^{-1}$). Neglect of these terms was further substantiated by momentum balance estimates of Lee and Pietrafesa (1987). Hydrographic data have indicated that changes in layer thicknesses are primarily due to horizontal advection, consequently we have ignored mixing across the layer interface. Finally, the model formulation does not allow for interface outcropping or the collapse of the lower layer. When this happens the computations must be terminated. Arbitrary bottom topography can be included by specifying depths at element nodes. Within each layer the vertically integrated equations of motion are solved in a Cartesian coordinate system with z positive upwards:

$$H_{i,t} + q_{ix,x} + q_{iy,y} = 0 \quad (1)$$

$$q_{ix,t} - f q_{iy} = -(F_{ip} - F_{ix}),_x + F_{iy,x} + \frac{1}{\rho_i} \{ \tau_{ix} - \tau_{(i-1)x} + p_i \xi_{i,x} - p_{(i-1)} \xi_{(i-1),x} \} \quad (2)$$

$$q_{iy,i} + f q_{ix} = -(F_{ip} - F_{iy})_{,y} + F_{ixy,x} + 1/\rho_i \{ \tau_{iy} - \tau_{(i-1)y} + p_i \xi_{i,y} - p_0 \xi_{(i-1),y} \}. \quad (3)$$

Subscript i is 1 or 2 for lower or upper layers, respectively; a subscript comma signifies a partial derivative with respect to the ensuing variables; H is layer thickness; u and v are fluid velocities; q_x and q_y are layer integrated volume transports; f is the Coriolis parameter ($=0.753 \times 10^{-4} \text{ s}^{-1}$ in the model runs); ρ the layer density where $\rho_1 = 1.0255 \text{ g cm}^{-3}$ and $\rho_2 = 1.0235 \text{ g cm}^{-3}$; ξ_0 is bottom elevation with ξ_1 the interface elevation and ξ_2 the surface elevation; p is pressure; τ the bottom, interface, or surface shear stress; F_{xx} , F_{yx} , F_{yy} are internal momentum exchanges; and $F_{ip} = \frac{1}{2} g H_i^2 + \rho_i^{-1} p_i H_i$.

The density difference reflects a temperature change of 6°C between surface and bottom layers. This density difference admits a baroclinic mode in the model and to some extent controls the internal wave characteristics. Mixing between layers is ignored and thus the densities remain constant at values typical of the hydrographic data (see Fig. 2).

The bottom and interface stresses are parameterized, respectively, as

$$\tau_{0x} = \rho_1 C_B (\bar{u}_1^2 + \bar{v}_1^2)^{1/2} \bar{u}_1 \quad (4)$$

$$\tau_{1x} = \rho_1 C_1 [(\bar{u}_1 - \bar{u}_2)^2 + (\bar{v}_1 - \bar{v}_2)^2]^{1/2} (\bar{u}_2 - \bar{u}_1) \quad (5)$$

with analogous expressions for the y -direction. The overbar variables represent the average layer velocity, $C_B = 2.3 \times 10^{-3}$ is the bottom friction coefficient, and $C_1 = 0.4 \times 10^{-3}$ is the interfacial shear stress coefficient. The value chosen for C_B is typical for bottoms with roughness heights of 20–30 cm and layer thickness of 30–40 m; similar values have been suggested by Thompson and O'Brien (1973) and Hickey and Hamilton (1980). Much less is known about the interfacial stress coefficient in oceanic flows. Average values range from 4×10^{-4} to 15×10^{-4} , Karelse (1974). Previously, O'Brien and Hurlburt (1972) have used the same value (0.4×10^{-3}) to study upwelling. The wind stress is parameterized in the usual way as

$$\tau_2 = \rho_{\text{air}} C_D |U_{10}| U_{10} \quad (6)$$

where ρ_{air} is the air density, U_{10} is the wind speed, measured at a height of 10 m MSL, and C_D is the wind stress drag coefficient, given by

$$C_D = (1.1 + 0.0536 U_{10}) \times 10^{-3}, \quad U_{10} \text{ in } \text{m s}^{-1} \quad (7)$$

as proposed by Wang and Connor (1975). This formulation is slightly different but in general agreement with that proposed by Wu (1980) and has been used successfully to model wind forced circulations in the same area during winter (Kourafalou et al., 1984).

The integrated internal stresses are parameterized as

$$F_{ix_k x_m} = E_{km} \left(\frac{\partial q_{ik}}{\partial x_m} + \frac{\partial q_{im}}{\partial x_k} \right); \quad i = 1, 2 \quad (8)$$

where $x_1 = x$, $x_2 = y$, and q_{ik} is the transport in layer i in the direction k , and E_{km} is an eddy viscosity tensor. The primary function of including the internal momentum transfer terms is to allow some dissipation of short period waves when these arise from the numerical calculations. These terms are set to zero unless specifically stated otherwise.

The system of equations is solved numerically using a finite element formulation with linear triangular elements for the spatial derivatives and a "split-time" finite difference scheme in time to advance the solution to the next time step. A time step $t = 300 \text{ s}$ is used in all model runs. A detailed description of these procedures can be found in Wang and Connor (1975) and Lorenzetti et al. (1986). The finite element grid used in the numerical experiments is shown in Fig. 3. It extends from Florida to North Carolina and from the coast to the shelf break at the 75 m isobath.

The model bottom topography is shown in Fig. 4. The observed thermocline depth is about 17 m below the surface (see Fig. 2), so that the model's interface is made to intersect the bottom at all grid points located in the row adjacent to the coast. The interface is allowed to move up and down at these points, which form a vertical wall, coastal boundary to the lower layer. In the model there is therefore only one layer in the row

FIG. 3. Finite element grid for the SAB. Number of nodes = 144 and number of elements = 231. Some of the nodes discussed in the text are indicated by their nodal numbers.

FIG. 4. SAB topography used in the numerical experiments. Contour in meters.

of elements closest to the coast and its depth is allowed to vary so that no discontinuity occurs at the inception of the lower layer.

At the coastal boundary, normal flow is set equal to zero for both layers and a slip condition is applied to the alongshore flow. For the specification of boundary conditions along the open boundaries, extensive testing showed that the best results were obtained by implementation of a sponge layer at the north and south extremes of the domain which extended from the cross-shelf boundaries to about $\frac{2}{3}$ of the regions south of Cape Canaveral and north of Cape Fear. These two layers effectively absorbed most of the wave energy near those boundaries. The rationale of the sponge layer implementation is explained in Lorenzzetti and Wang (1986). Adiabatic boundary conditions, i.e. fixed surface and interface elevations, were used along the offshore boundary. A sponge-layer implementation of the offshore boundary was rejected in favor of reducing computational cost. Because the calculated fields are numerically distorted and therefore have no physical meaning inside the sponge layers, results are not shown there.

3. Model experiments

a. Impulsive wind stress experiment

In this experiment a steady state wind stress field was applied at time $t = 0$ when all velocity and elevation fields were zero. All model integrations were carried out with a time step of 300 s. The objectives of this experiment were to estimate the frictional adjustment time of the SAB to a sudden change in wind forcing, to determine how the flow adjusts to this forcing, and to assess the dominant terms in the momentum balance; the eddy viscosity terms were set to zero.

The wind stress field in this experiment is shown in Fig. 5, where the "sticks" point in the direction towards which the wind is blowing. It represents a typical northward wind event in the SAB during the summer season. The interpolation scheme described in Allender (1977) was used to generate the wind stress at all grid points, employing the mean of observed summer winds

FIG. 5. SAB wind stress field generated from mean summer coastal and buoy winds with wind speeds increased by a factor of 10 to represent a typical summer wind event used for the impulsive wind stress experiment.

FIG. 6. Sea level time series at nodal points along the coast from the impulsive wind experiment. Number of each nodal point is indicated to the right of the plot.

at coastal stations of Daytona Beach, Jacksonville, Savannah, Charleston, and Wilmington, and at the NOAA meteorological buoys located at the shelf break, close to moorings 10 and 22, as shown in Fig. 1. In order to bring the coastal wind speeds to values that should be more representative of the wind speeds observed over the shelf, all coastal wind speeds were multiplied by 2, following a suggestion given by Kourafalou et al. (1984). This correction factor was determined by comparisons made between wind data collected at the Savannah airport and at the Savannah Lighthouse 16 km offshore. After the interpolation, the wind stress at each grid point was multiplied by 10 to better simulate a typical wind event since the mean wind stress over the region for the summer is considerably smaller than its standard deviation field.

1) SEA SURFACE ELEVATION

Time series of modeled sea surface elevations at different grid points along the coastline are shown in Fig. 6. As expected from a northward wind stress, a setdown of coastal sea level is observed as a consequence of the shelf response to the northward wind stress. Similar to the Beardsley and Haidvogel (1981) results for the Middle Atlantic Bight (MAB), our simulations also show that transient response of the SAB to an along-shore wind field is dominated by friction and rotation (momentum balances are given later). The model results indicate that the time required for the sea surface height field to reach a quasi-steady state is dependent on the position along the coast. Considering each one of the curves as an exponential function, the e -folding time, or spinup time is seen to be of the order of 12 to 24 hours. These values are roughly twice that observed by Beardsley and Haidvogel (1981) for the MAB (10–12 h), which may be due to the much weaker wind stress field observed during the summer over the SAB. A positive coastal longshore sea level slope of the order of $+1 \times 10^{-7}$ is suggested by the equilibrium levels at

nodes 92 ($\xi_2 = -8$ cm) close to Savannah and at node 50 ($\xi_2 = -12$ cm) north of Daytona Beach, separated by an approximate distance of 370 km. These nodes are in the vicinity of the current meter stations used by Lee and Pietrafesa (1988) to compute momentum balances from GABEX-II data. From their analysis an estimated mean alongshore sea level slope of $+0.7 \times 10^{-7}$ was deduced for midshelf, which compares very well with our model results. Figure 7 shows that the largest subtidal coastal sea level amplitude from the wind forcing tends to occur near the center of the domain where the shelf is widest. Lee et al. (1984) report finding the greatest coastal sea level amplitudes ranging from +10 to 30 cm at Savannah for the winter months. A wind set up towards the north is clearly indicated in Fig. 7 by sea level height lines which intersect the coast. The wavy spatial variability observed near the southern boundary is due primarily to the zero surface elevation condition applied there and a small amount of reflection from the sponge. The alongshore pressure gradient appears to be a result of wind setup and not due to the open boundary conditions of the model, because transient tests to be reported on in a following section show a reversal of the alongshore pressure gradient with the reversal of the wind forcing with constant boundary conditions.

FIG. 7. Free surface elevation contour for the impulsive wind experiment. Dashed lines indicate negative values. Contours in cm.

2) INTERFACE ELEVATION

The interface field obtained after one day of integration is displayed in Fig. 8. As expected from an upwelling favorable northward wind, positive interface anomalies are observed. Unlike the free surface field, the interface field is dominated by a series of patches, or centers of positive anomaly (positive vertical velocity). Observe that the two most intense upwelling cells are located around Cape Canaveral and close to Daytona Beach, where past research (Green, 1944; Taylor and Stewart, 1959; and Leming, 1979) shows that northward winds appear to drive a localized upwelling.

3) THE FLOW FIELD

Figure 9 shows velocity plots for upper and lower layers after two days of integration. As expected from the coastal jet dynamics, upper layer longshore flow shows a maximum at the coast, decreasing monotonically in the offshore direction. The upper layer flow tends to follow the curvature of the coastline with an offshore component, which seems more pronounced at the mid- to outer shelf regions. The upper layer velocity vectors in the north part of the domain tend to decrease and turn offshore as Cape Fear is approached. This seems to be related to the increased bottom friction and the topographic steering due to the shoaling present at the Cape. In order to compensate for the general offshore Ekman transport in the upper layer, Fig. 9 shows that the lower layer flow has an onshore component over most of the domain. Again, as expected from the two-layer coastal jet dynamics, in the lower layer, both the longshore and cross-shore velocity components tend to zero as the coast is approached. An interesting feature of the lower layer flow is observed close to Cape Fear where currents show a cyclonic circulation, with a bottom current that flows against the northward wind close to the shore. This seems to be caused by the negative alongshore pressure gradient induced by the positive surface setup along the coast. A comparison of upper and lower layer model velocities from the mid- to outer shelf indicates that the barotropic component of the flow is dominating the dynamics and only a small vertical shear is present. However, upper and lower layer flow in the region where the interface intersects the bottom (depth 20 m) show that a strong vertical shear (high baroclinicity) is present in that region. As expected, this band of strong baroclinic flow is trapped within a horizontal distance approximately equal to the internal radius of deformation, on the order of 8 km for the model parameters used.

4) MOMENTUM BALANCES

Figures 10 and 11 show the momentum balances for node 77, which is in the midshelf barotropic region. It is observed that upper layer x -momentum equation is dominated by a balance between the Coriolis accel-

FIG. 8. Interface elevation field after two days of integration for the impulsive wind experiment. Dashed lines represent negative and solid lines non-negative values. Contour in meters.

eration induced by the alongshelf flow and the cross-shore pressure gradient, with a small contribution of the wind stress in the cross-shore direction. The interface stress, as expected for the barotropic region, is negligible. The upper layer y -momentum shows a balance of the Coriolis acceleration induced by the offshore flow, the alongshore wind stress and the alongshore pressure gradient. Figure 11 shows that the lower layer cross-shelf momentum balance is dominated by an almost perfect geostrophic balance. Interface and bottom stresses play no role in the cross-shore dynamics at this point of the domain. A completely different picture is seen for the lower layer alongshore momentum balance. Due to the high barotropicity of the flow, the bottom y -flow is still strong enough to make the bottom stress dynamically important. The main balance in the y -direction is now between the bottom stress and the pressure gradient. A balance between Coriolis and local acceleration, typical of purely inertial motion induced by the impulsive start, is clearly seen in the y -momentum balance. This inertial oscillation has a period of about one day as expected at the 31°N latitude chosen for the model. Figure 12 shows the lower layer y -momentum balance for node 69, which is right at the baroclinic shear zone. At this location the interfacial

FIG. 9. Upper and lower layer velocity plots for the impulsive wind experiment. Time = 2 days.

stress can increase until it approximately balances the bottom stress. Finally, Fig. 13 for node 68 at the coast shows that for these shallow nodes the main balance in the alongshore direction is between the alongshore wind stress and the bottom stress. In the cross-shore direction, right at the coast, the cross-shore flow vanishes so that the momentum balance (not shown) is between the Coriolis acceleration of the along-shelf flow and the cross-shore pressure gradient.

b. Transient wind event experiment

Although the data show summer wind forcing to be highly coherent over the SAB, significant phase lags can occur. This indicates that the typical length scale of a summer wind event is similar to the length of SAB and that the propagation of the wind field is important. To investigate these effects and the associated dynamics, a period is selected for which the flow field is hind-casted.

The nine day interval from 0600 UTC 23 June 1981 to 0000 UTC 2 July 1981 was chosen because it contains a marked change in the wind stress field which starts northward, shifts towards the south, and then returns back to north at the end of the experiment. Interpolated nonuniform and nonstationary wind stress fields derived from observed winds were used based on the same wind interpolation scheme as before. The observed winds are shown in Fig. 14 and examples of the interpolated fields are shown in Fig. 15. Model parameters are also the same as before, except that initial tests resulted in premature outcroppings of the interface which had to be controlled by the addition of internal stresses. Closer inspection of these outcroppings showed that they were caused by very short (two grid length) oscillations superimposed on much larger anomalies. The short oscillations are almost entirely due to the propagation characteristics of the numerical scheme which in fact does not propagate waves of length equal to two grid spacings. This is a common feature in numerical models and is usually controlled, when needed,

FIG. 10. Upper layer x and y -momentum balances at node 77 for the impulsive wind experiment. Units are $\text{m}^2 \text{s}^{-2}$. l = local acceleration term, c = Coriolis term ($-fv$ and $+fu$ for the cross-shelf and along-shelf directions), p = pressure gradient term, w = wind stress term, and i = interface stress term.

by including only sufficient eddy viscosity to damp out these short waves, while, ideally the longer wavelength features are practically undisturbed. There may of course be physical features of very short wavelength, however, they cannot be resolved accurately in a grid which has only 2 grid spacings per wavelength. In this study we are focusing on elevation anomalies of greater wavelength, say four to five grid spacings, and the eddy viscosity used was $5000 \text{ m}^2 \text{s}^{-1}$. For a significant wave with a length of four grid spacings the eddy viscosity term is only about 3% of the largest terms in the momentum balance.

1) COASTAL SEA LEVEL

Figure 16 shows the 40-hour low-passed sea level fluctuations observed at Myrtle Beach (SC), Savannah (GA), and Daytona Beach (FL), together with model values at the corresponding nearest nodal points. These

sea level stations were chosen since they cover most of our study area with the two extreme stations being separated by about 600 km. A reasonably good agreement between predicted and observed coastal sea level is observed for the nine days of simulation. The initial mismatch is due to the spin-up process of the model, which was started from rest (zero elevation) and is only included in the plot for completeness. According to the steady wind test the spinup time is about 18 h, which agrees well with observations (Lee and Pietrafesa, 1988). After 18 h, the variations of observed and modeled sea level agree, however, the model sea levels appear high until about day two, because the model was started at too high a water level. All the troughs and ridges of the coastal signal were predicted by the model with little phase error. One exception to this is seen, however, at the first trough observed at Myrtle Beach, where the model leads the observed by about one day.

Both modeled and observed coastal sea level fluctu-

FIG. 11. Lower layer x and y -momentum balances at node 77 for the impulsive wind experiment. Same units and convention symbols of Fig. 10. b = bottom stress term.

FIG. 12. Lower layer y-momentum balance at node 69 for the impulsive wind experiment. Same units and conventions symbols of Fig. 11.

tuations had amplitudes of 20 to 30 cm and show a coherent southward propagation through the domain as indicated by the inclined dashed lines in Fig. 16 connecting the corresponding troughs and ridges. The average phase speed calculated from these plots by dividing the distance between the stations by the time delay of arrival of the coastal signal is about 500 km day^{-1} . During the nine days of simulation the coastal sea level fluctuations contained three southward propagating disturbances, giving an average period of 3 days as the dominant period. Coherent sea level fluctuations in the SAB with periods around three days have been observed in the past, Mysak and Hannon (1969) and Brooks (1979), and were considered forced by southward propagating meteorological disturbances. For the

FIG. 13. Alongshore momentum balance at the coast for node 68 and for the impulsive wind experiment.

FIG. 14. Time series of 6-hourly, rotated, 40-h low-passed wind vectors observed at five coastal stations from 23 June to 2 July 1981 used in the wind event experiment. Vertical sticks point northward.

modeled period presented here it appears that the subtidal coastal sea level fluctuations were basically responding to the local wind forcing mechanism.

A quantitative comparison between computed and observed elevations was performed ignoring the first two days of model spinup (Table 1). A total of 29 data points were used. Difference refers to observed minus predicted values.

The mean of the difference indicates that the model on the average is predicting elevations 1 to 2 cm higher than observed. This is a small deviation and could be corrected by lowering the prescribed elevation at the open boundaries. The rms values of difference are smaller than rms values of observations, indicating the degree to which the model is predicting the elevation better than a constant elevation prediction. Finally comparison of rms of difference with total range of variation provides an estimate of mean relative error. A large portion of this mean error can be seen in Fig. 16 to be due to slight phase differences between model and observations during wind shifts. A number of other causes are possible, such as the neglect of influences from river runoff at the coast and Gulf Stream fluctuations at the shelf edge, and the use of an interpolated wind field.

FIG. 15. Time series plots of the interpolated wind stress field used in the wind event experiment.

2) CURRENT VELOCITIES

Calculated and observed currents are displayed in Fig. 17. Due to the lack of current meter data in the inner shelf (0–20 m), the model results could not be verified for that region. At the shelf break no comparison was attempted since data analysis as well as previous work (Lee and Pietrafesa, 1988) have shown that the circulation in this region is primarily controlled by Gulf Stream mean currents and transient eddies and

meanders, which are not accounted for in the model. Comparisons in Fig. 17 are presented for the bottom current meter at mooring 3 and mid-depth meters at moorings 2 and 8 (see Fig. 1). The large northward currents at moorings 2 and 8 from 6/23 to 25 June are due to a major Gulf Stream induced upwelling tongue, which moved onto the shelf south of the moored array and extended northward at midshelf (Atkinson et al., 1988). Since the model was forced by wind only, it did not reproduce this feature. Difference statistics, means

FIG. 16. Comparison between modeled (solid line) and observed (crosses) coastal sea level for the wind event experiment. Slanted dashed lines were used to indicate southward propagation ($c \approx 500$ km day⁻¹) of coastal sea level.

and rms values of x - and y -component differences and rms speed difference between observed and computed velocities, are calculated for the last seven days of the model period (Table 2). In order to provide an estimate of how well the model is reproducing directions as well as speeds, the statistics are repeated with the model results rotated in 5° increments. From Table 2 it is seen that at mooring 3 the rms speed difference is smallest, 9.38 cm s⁻¹, when model results are rotated 5° counterclockwise, however effect of rotation between 5° and 15° is less than 0.5 cm s⁻¹. At moorings 2 and 8 the smallest speed differences, 14.98 cm s⁻¹ and 8.99 cm s⁻¹, are obtained at rotations of 20° and 15° , respectively. The reason for the discrepancy between model and observed directions for moorings 2 and 8 appears to be a result of the remaining effects of the previously mentioned cold subsurface Gulf Stream intrusion that entered the shelf south of our array (Atkinson et al., 1988).

TABLE 1. Comparison of observed and predicted sea levels for the last seven days, shown in Fig. 16.

Station	Difference (cm) (Observed - Predicted)		Observations (cm)	
	Mean	Rms	Rms	Range
Myrtle Beach	-1.00	5.88	7.25	32
Savannah	-1.11	5.84	12.50	38
Daytona Beach	-1.95	6.20	9.88	31

FIG. 17. Comparisons between modeled and observed currents for the wind event experiment. The numbers at the right of figures give the current meter numbers (see Fig. 1) and between parenthesis are given the depth of the meter and the total water depth. Vertical sticks point to north.

3) THE INTERFACE ANOMALY FIELD

Verification of the model solution for the interface position is a difficult task. The real water column is not exactly divided into two distinct layers as hypothesized in the model. Also the available hydrographic sections of temperature lack the temporal resolution to track the vertical position of the thermocline over time in a way which could be compared to model results. Although the numerical model does not solve for the temperature field, there is a well-known correlation between temperature fluctuations and the vertical velocities. Positive vertical velocity associated with cooling/upwelling events are represented in two-layer model by positive displacement of the interface. Negative vertical velocities are normally associated with warming/downwelling events and negative values of

TABLE 2. Comparison of differences (diff) between observed and predicted currents.

	Angle (deg)	X-diff (cm s ⁻¹)		Y-diff (cm s ⁻¹)		Speed difference (cm s ⁻¹)
		Mean	Rms	Mean	Rms	Rms
3 $\left(\frac{37}{40}\right)$	-30	1.6	8.54	1.3	6.94	11.01
	-25	1.3	7.79	0.9	6.93	10.43
	-20	0.9	6.98	0.4	7.10	9.96
	-15	0.5	6.14	0.0	7.40	9.61
	-10	0.0	5.31	-0.3	7.78	9.42
	-5	-0.4	4.56	-0.6	8.19	9.38
	0	-0.9	4.01	-0.9	8.61	9.49
	5	-1.4	3.82	-1.1	8.99	9.76
	10	-2.0	4.10	-1.3	9.31	10.17
2 $\left(\frac{17}{28}\right)$	-30	-0.4	6.99	0.7	13.35	15.07
	-25	-0.6	7.13	0.5	13.09	14.91
	-20	-0.9	7.41	0.3	12.91	14.88
	-15	-1.2	7.83	0.1	12.78	14.99
	-10	-1.5	8.39	-0.1	12.69	15.22
	-5	-1.8	9.09	-0.3	12.64	15.57
	0	-2.1	9.89	-0.4	12.61	16.03
	5	-2.4	10.80	-0.5	12.59	16.58
8 $\left(\frac{17}{28}\right)$	-25	-0.9	3.97	2.0	8.45	9.34
	-20	-1.2	3.74	1.9	8.30	9.10
	-15	-1.6	3.73	1.8	8.19	8.99
	-10	-1.9	3.99	1.8	8.12	9.05
	-5	-2.2	4.48	1.8	8.08	9.24

Note: A negative angle corresponds to a counterclockwise rotation of the model velocity vector.

the interface displacement. Bottom temperature records from current meter moorings were available during the modeling period, so that the interface elevation results can be compared against warming and cooling events present in these data.

During the nine days of simulation the two strongest signals present in the modeled interface elevation field were an upwelling event followed by a strong downwelling with their strongest magnitude located near Cape Canaveral (Fig. 18). These two events were related to a northward wind event followed by strong southward winds (Fig. 14). One sees from Fig. 18 that near Cape Canaveral, an upwelling cell develops from 26 to 27 June. The wind field then shifts to south and the upwelling cell decays and is replaced by a strong downwelling patch. The continuation in time of this figure (not shown) shows that the interface set down is very intense. In accordance with these results it can be concluded that during this period, the area near the Cape should exhibit first a moderate cooling followed by a stronger warming.

Figure 19 shows bottom temperature time series collected at moorings 2, 3 and 4, close to the upwelling region indicated by the model. The same trend indicated by the model is present in the field data: cooling from 24 to 27 June and warming after 28 June. Note that the temperature data were obtained a little north of the region where the maximum upwelling has been

historically observed. Because a vertically homogeneous water mass with temperature increasing away from shore would respond exactly opposite, we conclude that Ekman upwelling response did occur as predicted by the model.

c. The alongshore pressure gradient forcing experiment

The two previous numerical experiments used zero free surface and interface fields as boundary conditions at the open boundaries. This condition has the effect of isolating the shelf circulation from the influence of the deep ocean. It is well known, however, that barotropic and baroclinic currents can drive part of the circulation over the continental slope and even extend their influence to the midshelf regions. Lee and Pietrafesa (1988) clearly show the strong influence of the Gulf Stream on shelf circulation, especially at the shelf break moorings. In what follows, the spatial and temporal details of the Gulf Stream are neglected, and the influence of the deep ocean over the shelf is investigated by a parameterization that hopefully will provide some qualitative information about this process.

Previous studies give support to a parameterization of offshore currents by imposition of an alongshore pressure gradient at the shelf break (Semtner and Mintz, 1977; Beardsley and Winant, 1979). Momentum balance analysis made for the SAB using current

FIG. 18. Interface elevation plots for the wind event experiment. Solid lines represent positive and dashed lines negative values. Contours in 1-meter intervals. The numbers below each plot represent the date and local time.

meter and hydrographic data indicates that the sea level slopes downward toward the north at the shelf break with a gradient of the order of -10^{-7} during all seasons (Lee and Pietrafesa, 1988; Lee et al., 1984). This result is similar to the negative slope reported by Sturges (1974) for the Gulf Stream in this area. The alongshore sea level slope of -1.6×10^{-7} determined by Lee and Pietrafesa (1988) for the shelf break is then applied

along the shelf break open boundary of the model domain. The same numerical grid and physical parameters used in the last experiment are repeated. The wind stress field is set to zero in order not to confuse the isolated effect of the alongshore slope applied at the shelf break. The model is integrated over 3 days; Fig. 20 shows the free-surface and interface elevation contours and upper and lower layer velocity plot at the

FIG. 19. Time series of bottom temperature at moorings 2, 3 and 4 during the modeled upwelling/downwelling event.

end of the experiment. One sees that the flow is down the pressure gradient everywhere. Small onshore and offshore velocity components are observed in the lower and upper layers, respectively; this being more manifest in the southern part of the domain. In the northern part, the flow is seen to follow the isobaths and turn offshore, following the shoaling off Cape Fear, a characteristic of geostrophic flows. A sea level drop of about -6 cm between the coast and shelf break is observed at the widest part of the domain (~ 120 km width). This sea level drop generates a cross-shelf sea level gradient of 5×10^{-7} , which is balancing an average alongshore geostrophic current $v = (g/f)\partial\zeta_2/\partial x$ of about 7 cm s^{-1} . Note that the alongshelf pressure gradient is confined over the outer shelf region where the bottom slope steepens (see Fig. 4). A negligible alongshelf sea level slope is observed at the coast. The same type of trapping of the alongshore pressure gradient over the slope was also obtained analytically by Wang (1982) showing the insulating effect of the continental slope against the pressure field imposed by the deep ocean.

An interesting feature of the model solution is observed in the interface elevation plot. Similar to the northward impulsive wind experiment, an upwelling cell north of Cape Canaveral is evident in this figure. An average vertical velocity of 3×10^{-3} cm s^{-1} is indicated by an interface rise of 5 m in two days. The generation of upwelling north of Cape Canaveral by forcing the model through the negative sea level slope indicates that a northward upwelling favorable wind field is not the only mechanism capable of inducing the summer upwelling in the area. A negative sea level slope at the shelf break, or equivalently, an alongshore northward flowing current increasing towards the shelf break is capable of producing the same response. Blanton et al. (1981) pointed out that the localization of the upwelling cell just north of Cape Canaveral can be due to the spreading isobaths, which can generate a divergence in the northward flow field and thus a localized upwelling.

4. Discussion of results

Although the momentum balances presented in section 3a(iv) have been produced for each layer separately, the main dynamical balances can be compared to the vertically integrated momentum balances reported by Lee and Pietrafesa (1988). Also, if the different terms in the momentum balance results presented before are divided by layer thickness, the same order of magnitude estimates are obtained. Their analysis was made using GABEX-II data from the 28, 40 and 75 m isobaths at $30^\circ N$. The predominantly geostrophic balance in the cross-shelf direction indicated by the model results is also verified in that field data analysis for the midshelf region. The main balance in the alongshore direction for both model results and field data is between the Coriolis acceleration due to the cross-shore flow, the alongshelf pressure gradient, the alongshelf wind and bottom stresses.

The baroclinic contributions to the pressure gradient terms observed at the shelf break and to a smaller degree at the 40 m isobath by Lee and Pietrafesa were not present in the model results since they were produced by the baroclinic mass field of the Gulf Stream which was not included in the model. The role played by the interface stress cannot be compared to the field data analysis since in the vertical integrated approach these terms do not show up. The model results indicate, however, that in the narrow baroclinic zone close to the 17 m isobath, where large interface displacements occur, the interface stresses might become important in the dynamics of the flow.

Steady state momentum balances generated from the model runs also indicate that near the coast where water depth is small and the water column is basically one layer, the role played by the wind and bottom stresses tend to dominate the dynamics in the alongshore direction. This result could not be verified due to lack of observations, however, it has been observed in the nearshore waters of the Mid-Atlantic Bight (Csanady, 1982).

The above discussion indicates that, at least for the inner and midshelf regions, the model dynamics match reasonably well the dynamics observed in data. Neglecting the nonlinear advection terms in the model seems to be a reasonable approximation for the inner and midshelf regions. The field data analysis indicates, however, that a nonlinear model should be used for the outer shelf region because of the dominating presence of the Gulf Stream. In the absence of the Gulf Stream, the inclusion of the advective terms in the model should produce only a minor improvement of the model results for that region.

The above results indicate that the vertically integrated momentum dynamics for the entire water column of a quasi-steady flow in the SAB could be explained by the following analytical model representing

FIG. 20. Free surface and interface elevation contours (upper panels) and upper and lower layer velocity plots (lower panels) generated by an along-shelf pressure gradient of -1.6×10^{-7} imposed at the shelf break.

a geostrophic balance of vertically averaged momentum in the cross-shore direction and a balance between Coriolis, pressure gradient, wind and bottom stress terms in the alongshore direction:

$$f\bar{v} = g \frac{\partial \xi_2}{\partial x} \quad (9)$$

$$f\bar{u} = -g \frac{\partial \xi_2}{\partial y} + \frac{\tau_{2y}(Y)}{H(x)} - \frac{\tau_{0y}}{H(x)} \quad (10)$$

together with a continuity equation

$$\frac{\partial(\bar{u}H)}{\partial x} + \frac{\partial(\bar{v}H)}{\partial y} = 0. \quad (11)$$

The time derivatives were neglected since the frictional adjustment time, estimated to be less than a day, is relatively small compared to the two dominant time scales of the wind forcing, on the order of 3.5–4 days and 10–12 days. Therefore the above model will apply for alongshore wind forcing that is basically steady for time periods longer than one day. The cross-shelf stresses can be neglected compared to the Coriolis term due to the alongshore flow. Both the curvature of the coastline and the inhomogeneities observed in the longshore wind stress indicate that the y -dependence of the wind forcing must be included.

If the bottom stress term is parameterized using linear friction, then the above equations are the same as those used by Csanady (1978) in his arrested topographic wave (ATW) model. This indicates that the ATW dynamics could explain most of the wind forced response of the inner to mid-shelf regions of the SAB.

5. Conclusions

The aim of this study was to test the feasibility of modeling the summer circulation of the SAB with a two-layer finite element model. A simulation using a steady state, mean summer wind stress field was carried out to estimate the frictional adjustment time, and to determine the dominant terms of the momentum balance. In agreement with previous studies this experiment indicated that the adjustment time of the shelf circulation to sudden changes in the wind forcing varies between 12 and 24 hours. The main balance in the cross-shore direction was geostrophic. For the alongshore direction, Coriolis acceleration, pressure gradient, wind and bottom stresses are all significant in the momentum balance. The same experiment produced an upwelling interface anomaly field which qualitatively matched the summer upwelling observed north of Cape Canaveral and off Daytona Beach.

The response of the model to a transient real wind was studied for a 9-day period event. Modeled coastal sea level and upper and lower layer currents compared relatively well against observed coastal sea level and midshelf currents. Outer shelf flows did not compare

well against model results since they are governed primarily by the Gulf Stream mean and eddy components, not accounted for in the model.

The momentum balance results from these numerical experiments, associated with short adjustment time compared to the time scale of the forcing, indicates that a quasi-steady state is achieved most of the time between forcing and the shelf circulation. The resulting dynamical balance seems to be very similar to the arrested topographic wave model of Csanady (1978).

Model results, for both wind and alongshelf pressure gradient forcing experiments, indicate that localized summer upwelling observed in the Cape Canaveral area is not determined by a single factor, but rather the combined effects of upwelling favorable winds and Gulf Stream influences. The Gulf Stream influence occurs as an alongshore pressure gradient imposed at the shelf break, and the occurrence of meanders and frontal eddies (Lee and Pietrafesa, 1988). Northward winds and a negative alongshore pressure gradient both cause northward currents over the shelf with an offshore component in the upper layer and onshore component in the lower layer (upwelling). These combined upwellings apparently become intensified just north of Cape Canaveral due to the diverging isobaths in that area. An additional mechanism, which can further enhance upwelling, is provided by the simultaneous occurrence of a Gulf Stream frontal eddy in the outer shelf.

Acknowledgments. The authors are grateful for the technical assistance provided by Elizabeth Williams who processed some of the current meter data, and Jere Green who typed the several drafts. We thank L. Pietrafesa for use of the bottom temperature data from mooring 2. Support for this study was provided by the Department of Energy (Contract DE-AS05-76EV05163) and Minerals Management Service (Prime Contract AAT851-CT1-25 to Science Applications International Corporation of Raleigh, NC, Subcontract 11-82029-11). The L. Pietrafesa data were obtained with support from the Department of Energy (Contract DOE-AS09-74-00902). The first author wants to acknowledge the financial support provided for him by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil.

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