

Spatial relations across scales in land change models

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Abstract: *Land changes are the result of a complex web of interaction between human and biophysical factors, which act over a wide range of temporal and spatial scales. In this paper we conceptualize spatial relations among geographic objects at different scales. We analyze two types of spatial relations: hierarchical, which handles the interaction of nested objects at different scales (countries, states and municipalities, for example); and a network-based relation, which handles action-at-a-distance types of interaction (market chains, for instance) of objects at different scales (such as farms in Central Amazonia to soybean market consumers at the global scale). We implemented such relations in the Terralib environment, in which they can be constructed using selected strategies, and then used in dynamic models. We exemplify the use of such concepts in a real-world case study in the Brazilian Amazonia. We conclude that combining hierarchical and network-based spatial relations provide a comprehensive conceptual framework to include top-down and bottom-up interactions and feedbacks in multi-scale land-change models.*

1. Introduction

Modelling land change involves the use of representations of interactions within the land use system to explore its dynamics and possible developments [Verburg, Eickhout and Meij 2008]. Models can also be used to project the impact of policy changes on the current land use trajectory [Pijanowskia, Brownb, Shellitoc et al. 2002]. Land changes are the result of a complex web of interaction between human and biophysical factors, which act over a wide range of temporal and spatial scales. At different scales of analysis different factors have a dominant influence on the land use system: at the micro scale, land use patterns may be determined by household structure and local biophysical constraints. At the regional level the distances to markets and regional climate variations may determine land use pattern. Regional dynamics impact and are impacted by local dynamics through *top-down* and *bottom-up* interactions [Verburg, Schot, Dijst et al. 2004]. Understanding processes of change from the local to the global scale and their impacts on the coupled human-environmental system is a main scientific challenge [Moran, Ojima, Buchmann et al. 2005].

Land change processes are also intimately linked to processes of globalization. Globalization is the growing and accelerated interconnectedness of the world in an economic, political, social and cultural sense. It increasingly separated places of

consumption from places of production, such that land systems cannot be adequately understood without knowing their linkages to decisions and structures made elsewhere. In this sense, understanding the role of networks is essential to understanding land-use structure [Verburg, Schot, Dijst et al. 2004]. Such networks can be physical, such as infrastructure networks, and logical ones, such as market chains, linking a certain location to distant consumption or influential sites. According to Becker [2005]: “*it is impossible today, more than ever, to understand what happens in one place without considering the interests and conflicting actions at different geographical scales*”.

The goal of this paper is to discuss the incorporation of such hierarchical and network spatial relations in multi-scale land change models. We consider no single model or scale can handle the complexity of interactions that influence land change. Multi-scale land change models have been developed to address these issues. Some multi-scale modelling approaches combine different spatial models at different scales, mostly simulating *top-down* influences [Verburg, Eickhout and Meij 2008]. *Bottom-up* interactions and scaling issues have started to be addressed by multi-agent systems [Parker, Berger, Manson et al. 2002], in which interactions among individuals can simulate the emergent properties of the systems. Most land use change modelling embody the notion of space as a set of absolute locations in a Cartesian coordinate system, thus failing to incorporate spatial relations dependent on topological connections and network fluxes. Current land change models often deal with spatial interactions over large regions using (transport) network analysis to compute driving factors representing travel times and distant to ports, markets, etc. In spite of the progress in multi-scale modelling and spatial interaction analysis, there is still a need for approaches and techniques to deal adequately with scaling interaction issues [Verburg, Kok, Pontius Jr et al. 2006]. Understanding the interactions between and across scales, and the effects of globalization on local land-change processes, will remain the research frontier of land use/land cover for the next decade.

Our work contributes in this direction. We conceptualize spatial relations among geographic objects at different scales. We analyze two types of spatial relations: *hierarchical*, which handles the interaction of *nested* objects at different scales (countries, states and municipalities, for example); and *action at a distance* that handles the interaction through a *network* (market chain, for instance) of objects at different scales (such as grid cells in Central Amazonia to wood market consumers at the global scale). We argue these spatial relations provide a comprehensive conceptual framework to include *top-down* and *bottom-up* interactions and feedbacks in multi-scale models. This paper is organized as follows. Section 2 discusses the conceptual definition of these multi-scale spatial relations, and presents the implementation of these concepts using the Terralib GIS library [Câmara, Souza, Pedrosa et al. 2000]. Section 4 exemplifies, using a real world case study in the Brazilian Amazonia, how the explicit definition of such *hierarchical* and *action at a distance spatial relations* allow the representation of *top-down* and *bottom-up* linkages in multi-scale models.

2. Spatial relations across scales in land change models

In this paper, we use the definition of scale given by Gibson et al [2000]: “*scale is the spatial, temporal, quantitative, or analytical dimension used to measure and study any phenomenon*”. All scales have extent and resolution. In the case of spatial scales,

extension refers to the dimension of the study area, and resolution to the measurement precision. At each spatial scale geographic objects may be differently represented. Examples of representation of such objects include: (a) area regions whose boundaries are closed polygons; (b) cellular automata organized as sets of cells, whose boundaries are the edges of each cell; (c) point locations in two-dimensional space. For simplicity, we refer to the representation of spatial objects as *Entities*.

Our goal is to conceptualize the spatial relations between pairs of *Entities* at different scales to allow a broad representation of *top-down* and *bottom-up* interactions in land change models. We discuss two types of spatial relations: hierarchical and relative space relations.

Several existing land change models are organized in *top-down* manner, in which a demand for change is spatially allocated according to cell suitability. This includes the above mentioned CLUE [Veldkamp and Fresco 1996] and CLUE-S [Verburg, Soepboer, Veldkamp et al. 2002], Dinamica [Soares, Cerqueira and Pennachin 2002] GEOMOD [Pontius, Cornell and Hall 2001], and Environmental Modeler [Engelen, White and Nijs 2003]. Such models use *Hierarchical spatial relations*, in which nested scales are combined, as exemplified in Figure 1. The Environmental Modeler uses three different scales. Economic models at the national and regional scales compute land requirements for different land uses, based on economic and demographic factors. These land requirements are then allocated in a regular grid using a cellular automata model at the local scale. The CLUE model framework consists of two components: a demand module, that projects the amount overall of change; and an allocation module, the spatial component that acts in two scales (a coarse and a fine resolution grid) to localize such changes, based on cell suitability.

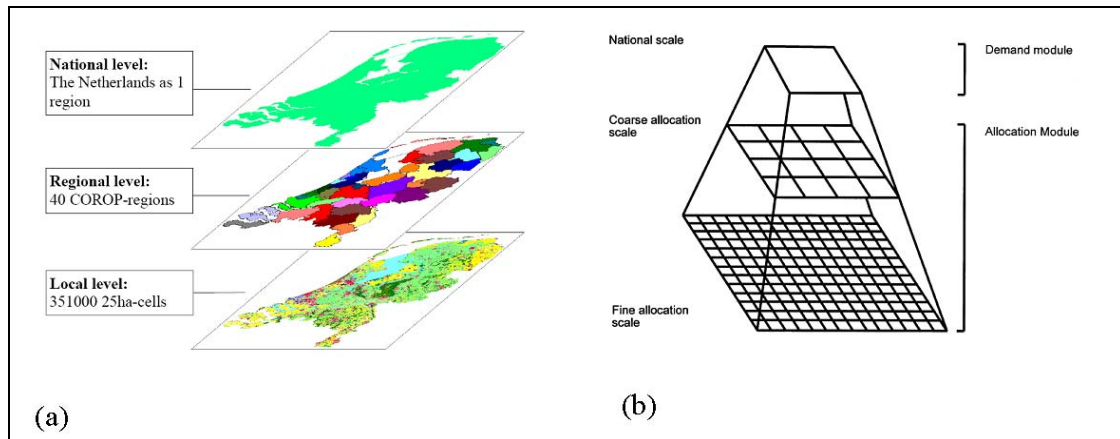


Figure 1. Examples of hierarchical structures used in land-change models: (a) Environmental Modeler [Engelen, White and Nijs 2003]; (b) CLUE model [Veldkamp and Fresco 1996].

Entities in these cases are regular cells with different resolution at different scales, or polygons representing administrative units at different levels of organization. The spatial relations represent parenthood relations (father-son and son-father). Father-son hierarchical relations are necessary to inform lower-scale model of the context provided by higher-level models, and are the most common type of relation found in current land-change models. Son-father relations allow local scale models to inform

regional models (*bottom-up* interactions). Although to some extent *bottom-up* interactions have been included in hierarchical land-change models (for example, Verburg et al. [Verburg, De Koning, Kok et al. 1999]), the full integration of *top-down* and *bottom-up* scale interactions is still a research topic [Verburg, Kok, Pontius Jr et al. 2006].

Such hierarchical spatial relations embody the notion of space as a set of absolute locations in a Cartesian coordinate system. However, flows of resources, information, organizational interaction and people are essential components of space, and should be treated in land change models. Efficient representation of such flows in connection with representation of absolute space is essential to achieve a realistic perspective of spatial relations, and inform land-change models [Harvey 1989]. These flows, which are normally represented as networks [Aguilar, Câmara and Cartaxo. 2003; Verburg, Schot, Dijst et al. 2004; Verburg, Kok, Pontius Jr et al. 2006], link processes that act on different scales. The global and continental market connections in Amazonia are an example of this, as Figure 2 illustrates. Different flows connect areas in the region to distance places of consumption, influencing the land use system in heterogeneous ways. Wood products from Brazil are mostly exported to Europe, as Figure 2.a illustrates. However, internal market also plays an important role in the wood market. Becker (2001) estimates about 80% is sold to the Southeast of Brazil. Global markets play a determining role for other commodities too. Santarém, in Pará State, has international connections related to the international soybeans markets, due to the presence of Cargill in the area. São Felix do Xingu, also in Pará, has different national and international connections related to the meat market, due to the presence of global companies like Bertin. The IRSSA (South-American Regional Infra-structure Integration Initiative, [IIRSA]) integration axes (Figure 2.b) will change the commercial connectivity of places like Roraima and Amapá, due to the Guiana-Venezuela-Suriname planned axe (Guiana Shield Hub). Large container transport companies, such as CMA-CGM, have already announced they will use the Madeira River corridor to export wood, cotton, and meat. The Madeira corridor is also part of Brazilian Infrastructure Plans for the Amazonia, linking Porto Velho, Rondonia State, to Manaus, in Amazonas State. Incorporating such heterogeneous connections in land change models is essential to improve our understanding about their impacts on the land use system, and to envision the future scenarios for the region.

Combining such hierarchical and network-based relations is necessary to provide the necessary conceptual support to multi-scale land change models. Sections 2.1 and 2.2 present a conceptualization of these two types of relations. Our implementation of such concepts is briefly described in Section 2.3.

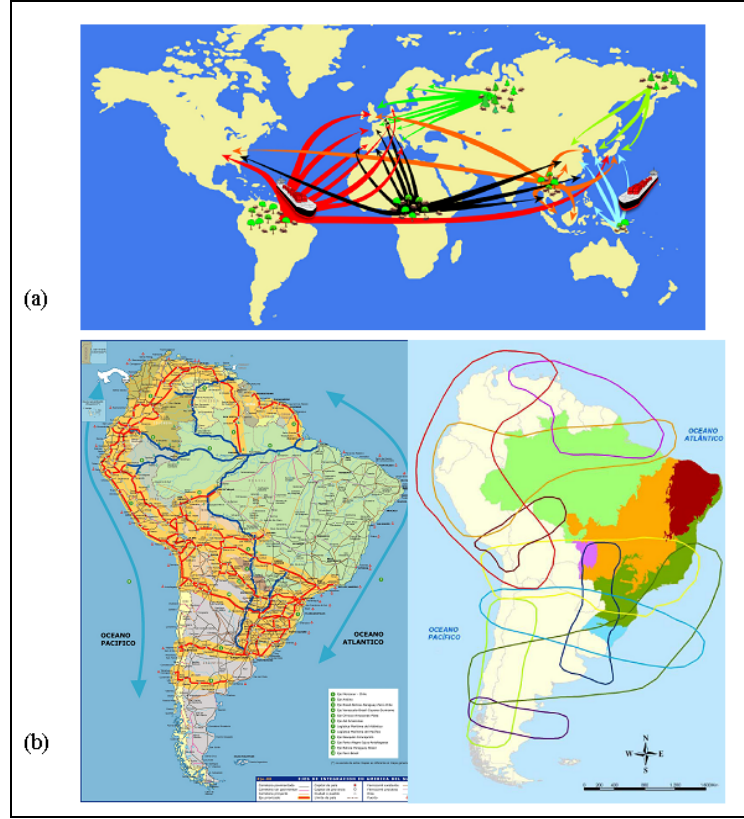


Figure 2. Examples of links to global and continental markets: (a) International flow of wood from Amazonia (source: Greenpeace, www.greenpeace.org); (b) IIRSA infra-structure integration axes in South America, facilitating the commercial flow of different areas to Europe, North-America and/or Asia markets.

2.1. Hierarchical relations

We propose to represent a hierarchical relation as a directed graph G composed of set of nodes N_1 and N_2 , representing *Entities* at $Scale_1$ and $Scale_2$; and a set of arcs A linking nodes N_1 to N_2 .

The arcs A can have attributes or not, depending on the strategy used to construct them. When *Entities* at both scales have an area representation (polygons or regular cells), we propose three alternative strategies, illustrated in Figure 3. They are based on topological relations as described below.

- *Simple*: when spatial resolutions are perfectly matched, simple “within” or “coveredby” or “equals” spatial operator can define the parenthesis relation between scales.
- *ChooseOne*: for area representations, when hierarchical spatial resolutions do not match, this strategy chooses the upper scale unit cells with larger percentage of intersection as the father and the “intersection” spatial operator can define the relation.

- *KeepInBoth*: also only for area representations, when hierarchical spatial resolutions do not match, this strategy keeps all intersected upper units cells as fathers and the “intersection” spatial operator can define the relation. The percentage of each intersection is stored as an attribute of the Arc A .

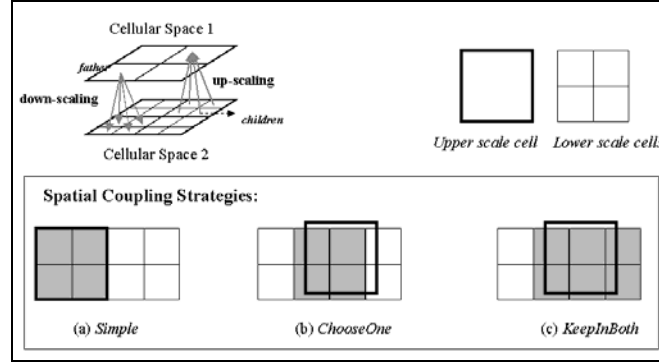


Figure 3. Schematic representation of strategies for spatial coupling in the case of regular cells: (a) Simple; (b) ChooseOne; (c) KeepInBoth.

Hierarchical networks can represent spatial relations of point entities at different scales, such as urban centers (State capital at the macro scale; major cities at the meso scale; villages at the local scale). To construct graph G in this case, manual or attribute based strategies could be envisioned (for example, administrative unit name to establish son-father relations). The attributes of the Arcs A could also be derived from geographical objects (such as percentage of population).

2.2. Network-based relations

We also represent *network-based relations* as a directed graph G composed of set of nodes E_1 and E_2 , representing *Entities* at Scale₁ and Scale₂, and a set of arcs A linking nodes E_1 to/from E_2 . The representation is the same as for hierarchical relations. The difference resides in the strategies to construct G . A network T is required to represent physical (roads, rivers, energy) and logical (airline routes, market chains, migration fluxes) linkages between elements E_1 and E_2 . These linkages will be established using network analysis operators.

According to characteristics of the network, specific construction strategies will decide: (a) if two nodes in E_1 and E_2 are connected; (b) the strength of this connection. The construction strategies presented here are based on the concepts introduced by Aguiar et al [2003] regarding the construction of a *Generalized Proximity Matrix* (GPM). The GPM represents *absolute and relative space neighborhood relations* among objects of the same type, at the same scale. A GPM is used to support spatial analysis and cellular automata dynamic models. We modify the GPM construction strategies to consider objects of different types, at different scales to support the development of multiscale land-change models. Two strategies are then proposed:

- *Multi-scale Closed-networks* linkages: to connect entities at different scales using networks in which the entrances and exits are restricted to its nodes. They encompass *logical* (such as banking networks and productive chains) and some types of *physical networks* (railroads, telecommunication networks).

- *Multi-scale Open-networks* linkages: to connect entities at different scales using networks in which any location is entrance or exit point. These are always *physical networks*. Examples are transportation networks such as roads and rivers. For open networks, it is necessary to make use of the actual line coordinates that correspond to each arc in order to be able to compute the closest entrance/exit points from any arbitrary position.

The strategies can be summarized as follows:

- For each object in O_1 , compute the nearest entry point E_1 in network T .
- For each object in O_2 , compute the nearest entry point E_2 in network T .
- The existence of a linkage from E_1 to/from E_2 is computed using network analysis.

Figure 4 illustrates the process of constructing graph G to represent relative space relations. A set of parameters bounds connectivity limits according to network and case study characteristics. For instance, one can define objects at $Scale_1$ are not linked to the network if they are more than a 100 km away from the closest entry point. Limits can also be imposed for minimum path in the network. For instance: only objects at $Scale_1$ not more than 10 hours from the markets (represented at $Scale_2$) through the infrastructure network are considered connected. Minimum path computation depends on network attributes. Different case studies can use, for example, distance or travel time (infrastructure networks), flow of people (migration networks), dollars (banking networks), added value (production chains).

Note that when *Entities* at both scales have an area representation (polygons or regular cells), the connection is performed using the area centroid.

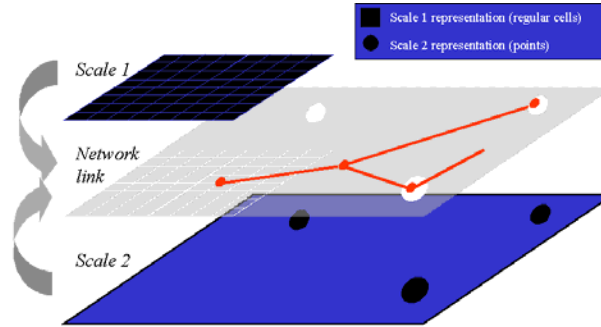


Figure 4. Schematic representation of a network-based spatial relation between cell objects in Scale 1 and point objects in Scale 2.

2.3. Implementation

We implemented the conceptual definitions presented in Sections 2.1 and 2.2 using the Terralib GIS library. For both types of relations, construction strategies were added to the library as an extension of the Generalized Proximity Matrix functionality [Aguiar, Câmara and Cartaxo. 2003]. The new strategies deal with relations among objects in two different layers of information, representing the geographic objects at different scales. The relations can be constructed for polygon, points and cell representations of objects. For hierarchical relations only the absolute space Cartesian coordinates are

considered to define the father-son and son-father relations. For network-based relations, a third layer is necessary representing a logical or physical network used to define the connectivity between the objects. The case study described below uses the Terralib implementation to construct the graphs G representing the relations. They are stored in a database, and can be exported as text files. Once constructed, tools for dynamic modelling can be applied using the relations. We use the TerraME modelling environment [Carneiro 2006] to develop our land-change models using the stored relations.

3. Example in the Brazilian Amazonia

We exemplify the use of concepts presented in the previous sections in a multiscale land change model for the Brazilian Amazonia developed by Aguiar [2006] and Moreira et al [2008], using the TerraME modelling environment [Carneiro 2006].

The model encompasses three scales: (a) at the national level, the main markets for Amazonia products (Northeast and São Paulo) and the roads infrastructure network; (b) at the regional level, a regular grid of $25 \times 25 \text{ km}^2$ resolution cells for the whole Brazilian Amazonia, covering an area of approximately 4 million km^2 ; and (c) at the local level, a nested regular grid of $1 \times 1 \text{ km}^2$ resolution cells for a hot-spot of deforestation in Central Amazonia, the Iriri region, in São Felix do Xingu, Pará State. This local grid covers an area of approximately 50,000 km^2 . Figure 5 illustrates the three scales and their geographic objects representation.

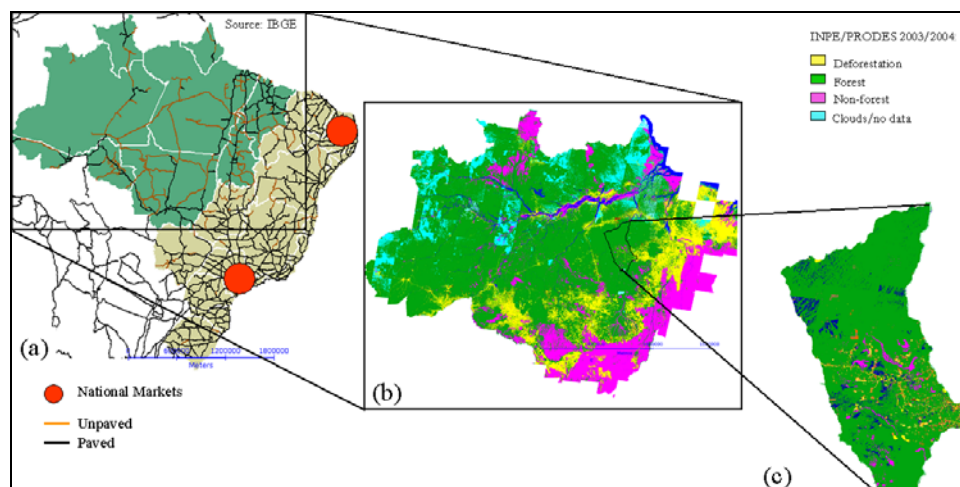


Figure 5. Study area: (a) Brazil: roads network and main markets (São Paulo and Northeast); (b) Brazilian Amazonia: deforested areas map; (c) Iriri/Terra do Meio in São Felix do Xingu, Pará State: deforested areas map.

The goal of the multi-scale model is explore *complementary information about the occupation process* in the region [Moreira, Costa, Aguiar et al. 2008]. The model includes the following interacting model components and spatial relations:

- The regional scale model projects the percentage of deforestation for each $25 \times 25 \text{ km}^2$ cells. We used a statistical allocation procedure based on regression models adapted from the CLUE model [Veldkamp and Fresco 1996] by Aguiar [2006]. It represents the process of agricultural frontier expansion over the whole Brazilian Amazonia. The macro model seeks to answer questions such as:

given a certain pressure for expansion of agricultural land, which areas in the Amazonia would be occupied first? One of the goals was to explore the hypothesis that connection to national markets through roads infrastructure is a key factor to explain the distribution of deforestation in the region. *This requires the establishment of a network-based relation to link the cells in Amazonia to places outside the region.* This relation is used in defining the suitability of the 25 x 25 km² cells for change according to their level of connectivity.

- The nested local model seeks to answer questions such as: given that a certain amount of pressure is projected for the Iriri by the regional model, how would local patterns of occupation evolve? The *top-down* interaction consists of the regional model signalling an expected demand for change at the Iriri. This demand is calculated as a function that sums-up the deforested area (father-cells) at the regional scale and sends it to the local scale. *This requires a father-son relation to select the 25 x 25 km² cells corresponding to the Iriri 1 x 1 km² cells.* The model uses this relation to add the large-scale projected change at 25 x 25 km² cells and send the resulting a demand for change to the local model.
- The Iriri model is an agent-based deforestation model [Moreira, Costa, Aguiar et al. 2008]. Two sets of agents were identified: small and large farmers. Small settlers favour proximity to roads and urban centres. Large farmer prefer large pieces of inexpensive land, not necessarily close to roads. Therefore, each type of actor is associated to set of determining factors and decision rules. Local policy decisions, expressed at local scale, may prevent the full extent of projected change from occurring. A *bottom-up feedback mechanism* sends this information back to the larger scale and thus modifies the macro scale model corresponding cells. *This requires a son-father relationship to link 1 x 1 km² cells to the upper-scale 25 x 25 km² cells.* The model used this relation to correct the projected change at the 25 x 25 km² cells.

To support the implementation of such scale interactions in this land-change model, we defined and computed the following hierarchical and network-based relations.

3.1. Hierarchical relation between the nested grids

We used a hierarchical relation to provide the spatial support to dynamically link the two nested grids at 25 x 25 km² and 1 x 1 km² resolutions. The strategy we use to construct the relation is the *KeepInBoth*, as the cellular spaces were not coincident.

Each coarse scale cell is linked to approximately 625 finer scale cells (father-son relation). Most finer scale cells are linked to only one coarser scale cells (son-father relation), but depending on their relative position (on the borders of the coarse scale cells) they can be linked to two, three or even four parent cells (see Figure 3.c). The father-son and son-father hierarchical relations allow the incorporation of *top-down* and *bottom-up* interactions between the regional and local models, as discussed in Section 2.1.

3.2. Network-based relation: connection to markets

We used an *Multi-scale Open-network* strategy to connect the regional scale 25×25 km² cells to the main places of consumption at the national scale (São Paulo and Northeast). Graph G representing the relation between these objects is computed using the following parameters:

- Maximum distance from cells to the road network: unbound (all cells are included).
- Maximum distance from entrance points E through the network: unbound.
- Weight computation: inversely proportional to the minimum path distance from the cell to each national market, using the roads network. We distinguished paved from non-paved roads (non-paved roads are supposed to double the distances).

Graph G includes the $2:n$ relationship from the two markets to every cell, and a $n:2$ relationship from every cell to the two markets. Both directions could be used in land-change models. For example, the $2:n$ (from market to cells) could be used to establish a remote influence between São Paulo and their most connected cells. We could include a rule in the model to bound change in Amazonia cells as a result of a behavioural or policy change in São Paulo. This change in the market conditions can be an incentive (demand increase) or a restriction (need of certification).

In this paper, the land-change model uses the $n:2$ relationship (from cell to market). We derive a new cell attribute based on graph G to represent the level of connectivity of each cell to any of the markets. If road conditions change, the variable is recomputed. Each cell receives as attribute *conn_markets* the minimum *weight* value stored in G according to the roads network at that time. Figure 6 illustrate the connection to markets variable in 1997 and the projected 2010 level of connectivity, supposing some roads are to be paved.

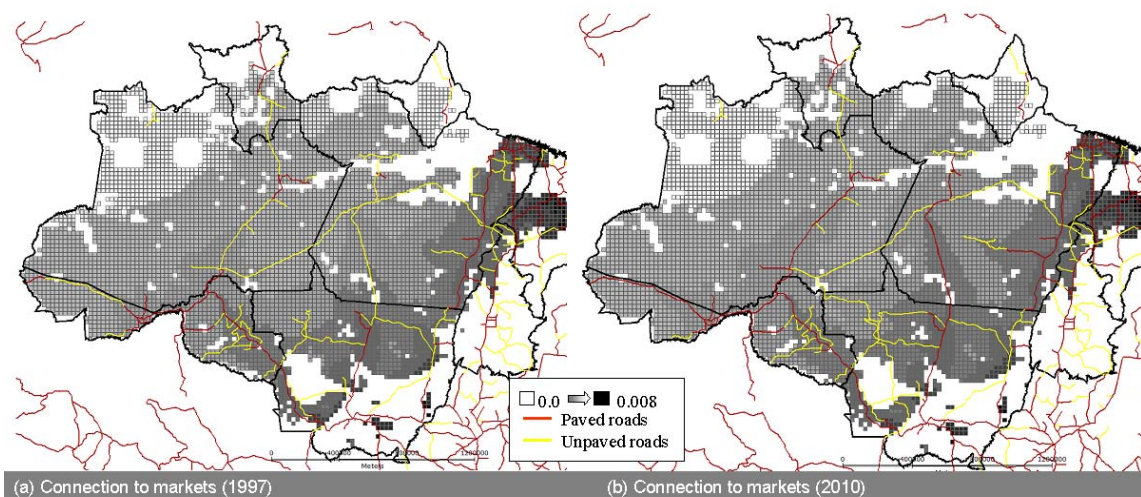


Figure 6. Connection to markets variable constructed using a network-based multiscale spatial relation: (a) in 1997; (b) in 2010 (paving some roads).

This network-based relation was used to construct one of the main variables in the model. Using the connection to national markets, the model was capable of reproducing the different stages of occupation of the new frontiers of the XXI century, using 1997 as the base year, including São Felix do Xingu [Aguiar 2006] comparing to 2003 deforestation maps (INPE, 2008). The model captures the process in which cattle ranchers decided to migrate to the São Felix area due to its biophysical, accessibility and market conditions. The connection to markets variable represents a process that acts in a higher hierarchical level, and could not be captured in a single scale study.

4. Conclusions

This paper discussed and conceptualized the use of multi-scale spatial relations in land change models. Two types of relations were presented: hierarchical and network-based. Multi-scale land-change models are often based on hierarchical relations, using nested objects at different scales. We argue that combining hierarchical relations with network-based relations provide a comprehensive conceptual framework to include *top-down* and *bottom-up* interactions and feedbacks in multi-scale land-change models. Network-based relations can represent remote influences in the land use system. This has a growing importance in a globalized economy, in which places of consumption and production are increasingly separated. Land systems cannot be adequately understood without knowing the linkages of different areas to decisions and structures made elsewhere. We exemplified the use of such relations in a multi-scale land change model for the Brazilian Amazonia.

Acknowledgements

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