

# COMPARING MORHOLOGICAL LEVELINGS CONSTRAINED BY DIFFERENT MARKERS

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8<sup>th</sup> International Symposium  
on  
Mathematical Morphology

# Subject...

"Comparing morphological levelings constrained by different markers"

## *★ Study the scale space image representations of Morphological Levelings*



Brief background:

- ★ ..from linear scale space representations, like Gaussian Isotropic ones (ISO)  
... which blur image edges
- ★ ..to non-linear scale space representations...

*i)* Anisotropic Diffusion

*ii)* Morphological Scale Spaces

# Background...

"Comparing morphological levelings constrained by different markers"

## Morphological Scale Spaces

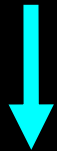
...from classical dilations & erosions



...which displace image edges

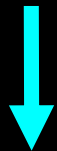
... to openings  $rec^+$  **RO** & closings  $rec^-$  by reconstruction

treat assymetrically image regions (image: foreground/ peaks and background/ valleys)



... to Alternating Sequential Filters, **ASF**)

$$f = rec^+ \left( rec^- \left( \dots rec^+ \left( rec^- \left( rec^+ \left( rec^- (orig \mid 1B) \mid 1B \right) \mid 2B \right) \mid 2B \right) \dots \mid rB \right) \mid rB \right)$$



computational exhaustive

... to Levelings...

# Background...

"Comparing morphological levelings constrained by different markers"

## Morphological Levelings

...introduced by

*Meyer F., 1998. From connected operators to levelings, Mathematical Morphology and Its Applications to Image and Signal Processing (H. Heijmans and J. Roerdink, Eds.), pp. 191–198, Kluwer Academic, Dordrecht/ Norwell, MA.*

...further studied by

*Matheron G., 1997. Les Nivellements, Technical Report, Centre de Morphologie Mathematique.*

*Serra J., 2000. Connections for sets and functions. Fundamentae Informatica, 41:147–186.*

...further studied and  
formulated via PDEs

*Meyer F. and Maragos P., 2000. Nonlinear Scale-Space Representation with Morphological Levelings" J. Visual Communic. and Image Representation, vol.11, pp.245-265.*

...applied for image  
segmentation tasks

*Meyer F., 2004. Levelings, Image Simplification Filters for Segmentation, J. of Mathematical Imaging and Vision, vol.20, pp. 59 - 72.*

# Background...

"Comparing morphological levelings constrained by different markers"

## Scale Spaces from Anisotropic Diffusion

*Weickert J., 1998. Anisotropic Diffusion in Image Processing. Teubner, Stuttgart, Germany*

...among various formulation those of

Perona and Malik, 1990

Alvarez, Lions and Morel, 1992 **(ALM)**

Black and Sapiro, 2000



have been studied and associated with levelings, **producing**  
simplified images with different characteristics

# Goal...

"Comparing morphological levelings constrained by different markers"

- ★ Study the resulting simplified images and
- ★ Conclude on:
  - ★ ... the characteristics of the each scale space representation
  - ★ ... which marker is most appropriate for image pre-processings or general edge detection, segmentation tasks

# Overview

- ★ Goal 
- ★ Background (...brief) 
- ★ Relation between scales (...an effort towards..)
- ★ Comparison:
  - ★ Qualitative evaluation
  - ★ Quantitative evaluation
- ★ Conclusions / Future perspectives

# Few notations...

*"Comparing morphological levelings constrained by different markers"*

## Morphological Levelings

**Levelings** are transformations, when one **marker** image ***h*** is transformed to a function ***g***, which is a **leveling** of the initial/reference image ***f***

$$g \wedge f \leq h \leq g \vee f$$

With the use of a **marker** ***h***, levelings can be constructed in the following way:

★ in cases where ***h* < *f***, values of ***h*** are **replaced** with the **min** of ***f, δh***  $f \wedge \delta h$

★ in cases where ***h* > *f***, values of ***h*** are **replaced** with the **max** of ***f, εh***  $f \vee \varepsilon h$



# Few notations...

"Comparing morphological levelings constrained by different markers"

## Morphological Levelings

**Levelings** are transformations, when one **marker** image  **$h$**  is transformed to a function  **$g$** , which is a **leveling** of the initial/reference image  **$f$**

$$g \wedge f \leq h \leq g \vee f$$

With the use of a **family of markers**  **$h_i$**   $\{h_1, h_2, \dots, h_n\}$

a scale space representation  **$g_i$**  can be constructed:

$$g_1 = f, \quad g_2 = \Lambda(f, h_1), \quad g_3 = \Lambda(f, h_2), \quad \dots, \quad g_{n+1} = \Lambda(f, h_n)$$

# Levelings Construction...

*"Comparing morphological levelings constrained by different markers"*

## Different type of markers:

Levelings are traditionally associated with isotropic (ISO) or other (RO), (ASF) morphological markers

★ Meyer, 1998; Matheron, 1997; Serra, 2000, Maragos, 2000; Meyer, 2004.

Recently, were associated with anisotropic diffused markers (ADF)

★ A. Sofou, G. Evangelopoulos, and P. Maragos. *Soil image segmentation and texture analysis: a computer vision approach*, IEEE Geoscience and Remote Sensing Letters, 2 (2005), no. 4, pp.1-4.

★ Karantzas K., Argialas D. and Georgopoulos A., *Anisotropic Morphological Levelings*, International Journal of Image and Vision Computing (under review, submitted: 08/2006)

# Anisotropic Morphological Leveling

"Comparing morphological levelings constrained by different markers"

$$g_i = \Lambda(I, f_i) = \Lambda\left(I, \left[\frac{\mathcal{G}I(x, y, t)}{\mathcal{G}t}\right]_i\right) = \Lambda\left(I, \left[r\left(\|G_\sigma * \nabla I\|\right) |\nabla I| \operatorname{div}\left(\frac{\nabla I}{|\nabla I|}\right)\right]_i\right)$$

★ where  $g_i$  is the AML of scale  $i$

*two scale parameters:*

★  $i$  Scale of leveling

★  $t$  Scale of anisotropic diffusion (PDE's iterations)

# Markers scale relation

"Comparing morphological levelings constrained by different markers"

Scale parameters:

*Towards the establishment of a scale relation ...*

★ ISO:  $\sigma$

★ RO:  $r$

★ ASF:  $r$

★ ADF:  $t$

Leveling's Scale	Values for the scale relation of the four different type of markers			
	Structure element's size $r$ for RO and ASF	Isotropic diffusion		Anisotropic diffusion iterations $t$
		Standard deviation $\sigma$	Kernel size	
1	1	0.5	$5 \times 5$	100
2	2	1	$7 \times 7$	200
3	3	1.5	$11 \times 11$	300
4	4	2	$13 \times 13$	400
5	5	2.5	$17 \times 17$	500
6	6	3	$19 \times 19$	600
7	7	3.5	$23 \times 23$	700

*Proposed values for scales: 1 to 7.*

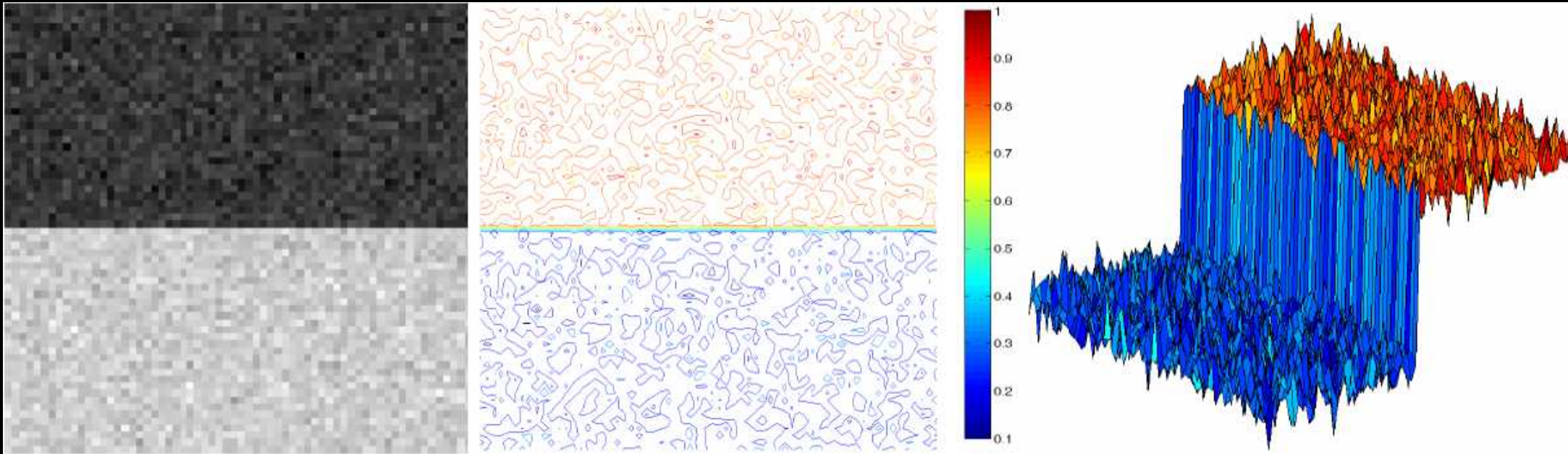
*After extensive experiments/applications, attempting to*

- i) understand the extent of their filtering effect and*
- ii) relate their result towards an approximate equivalence*

# Evaluation

"Comparing morphological levelings constrained by different markers"

*artificial test image: a binary 'chessboard type'  
with both additive and salt and pepper noise*



*Test image*

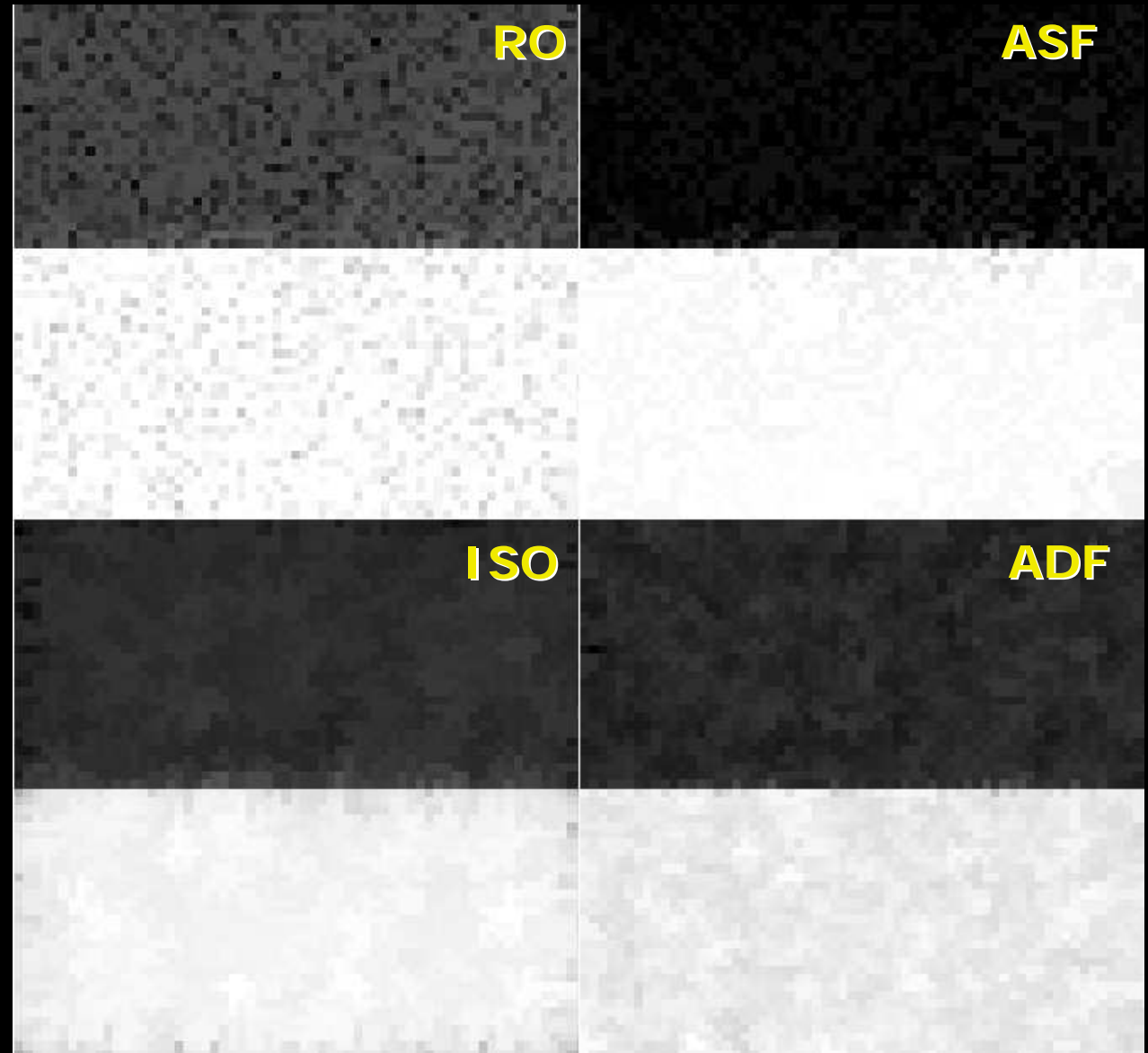
*contours-isophotes*

*3D representation where image  
brightness values are proportional to  
surface height*

# Evaluation

"Comparing morphological levelings constrained by different markers"

- ★ **All** methods did not displaced edges
- ★ **ASF** marker leveling differed most from the reference image intensities



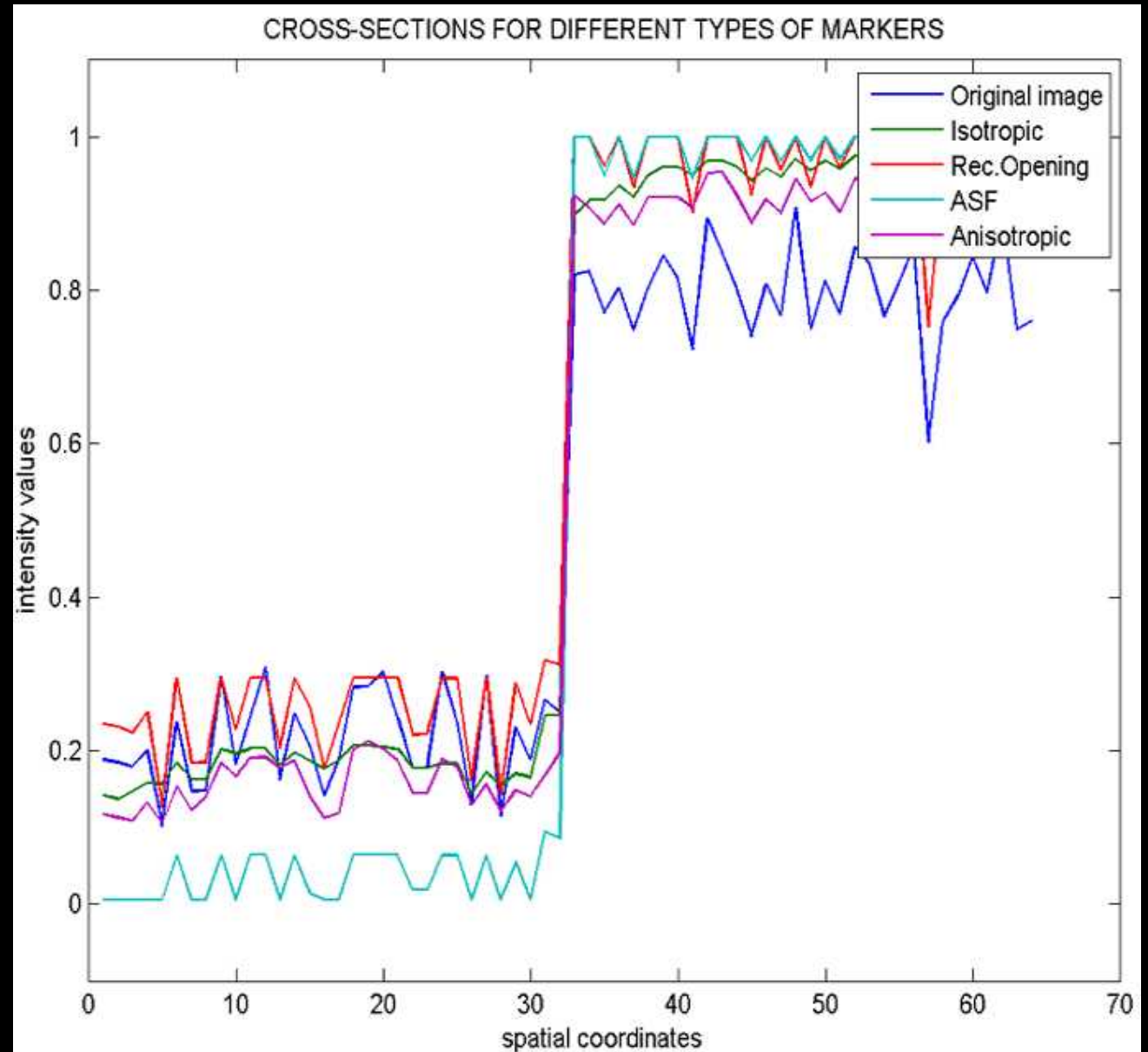


# Evaluation

*"Comparing morphological levelings constrained by different markers"*

★ **ISO** marker simplified but at the same time smoothed intensity difference between image's zones.

★ **ADF** marker simplified and at the same time followed, more constantly, both values and changes in image intensity values.



# Quantitative Evaluation

"Comparing morphological levelings constrained by different markers"

...focused on measurements assessing

★ the extent of intensity variation (RMSE & NMSE) and

★ the structural similarity (SSIM)

between the reference image and the resulting leveling

		Markers			
		RO	ASF	ISO	ADF
Test image scale 2	RMSE	0.126	0.187	0.118	0.103
	NMSE	0.046	0.101	0.040	0.031
	SSIM	0.9981	0.9956	0.9984	0.9989

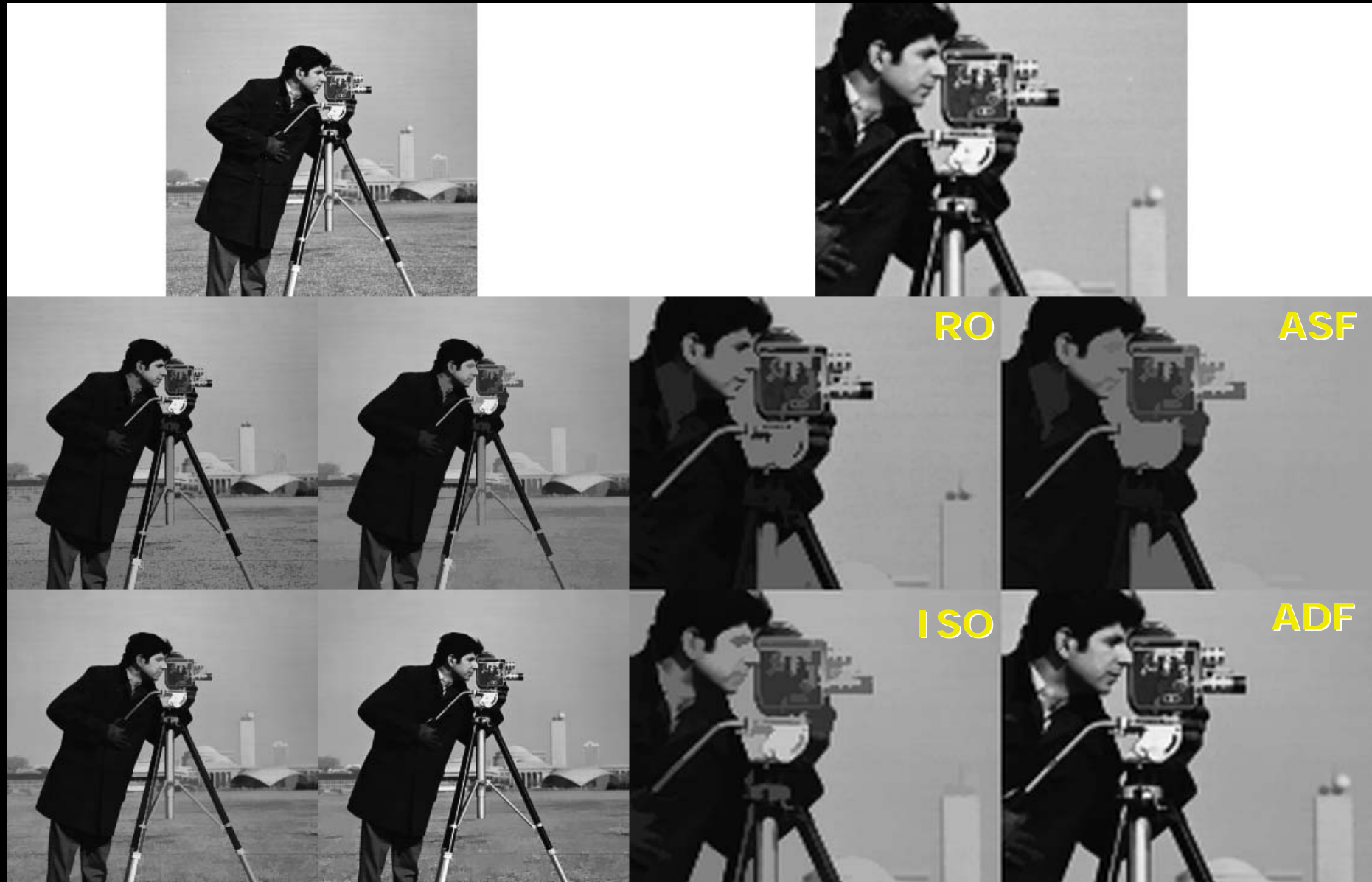
★ **ASF** marker yielded to the larger RMSE, NMSE values (its brightness values differ much from the original image) and to the smallest SSIM value, that confirms its lower structural similarity with the reference image.

★ **ADF** marker kept the closest relation with the reference image, regarding both i) the extent of intensity variation (RMSE and NMSE) and ii) their structural similarity (SSIM).



# Evaluation

"Comparing morphological levelings constrained by different markers"



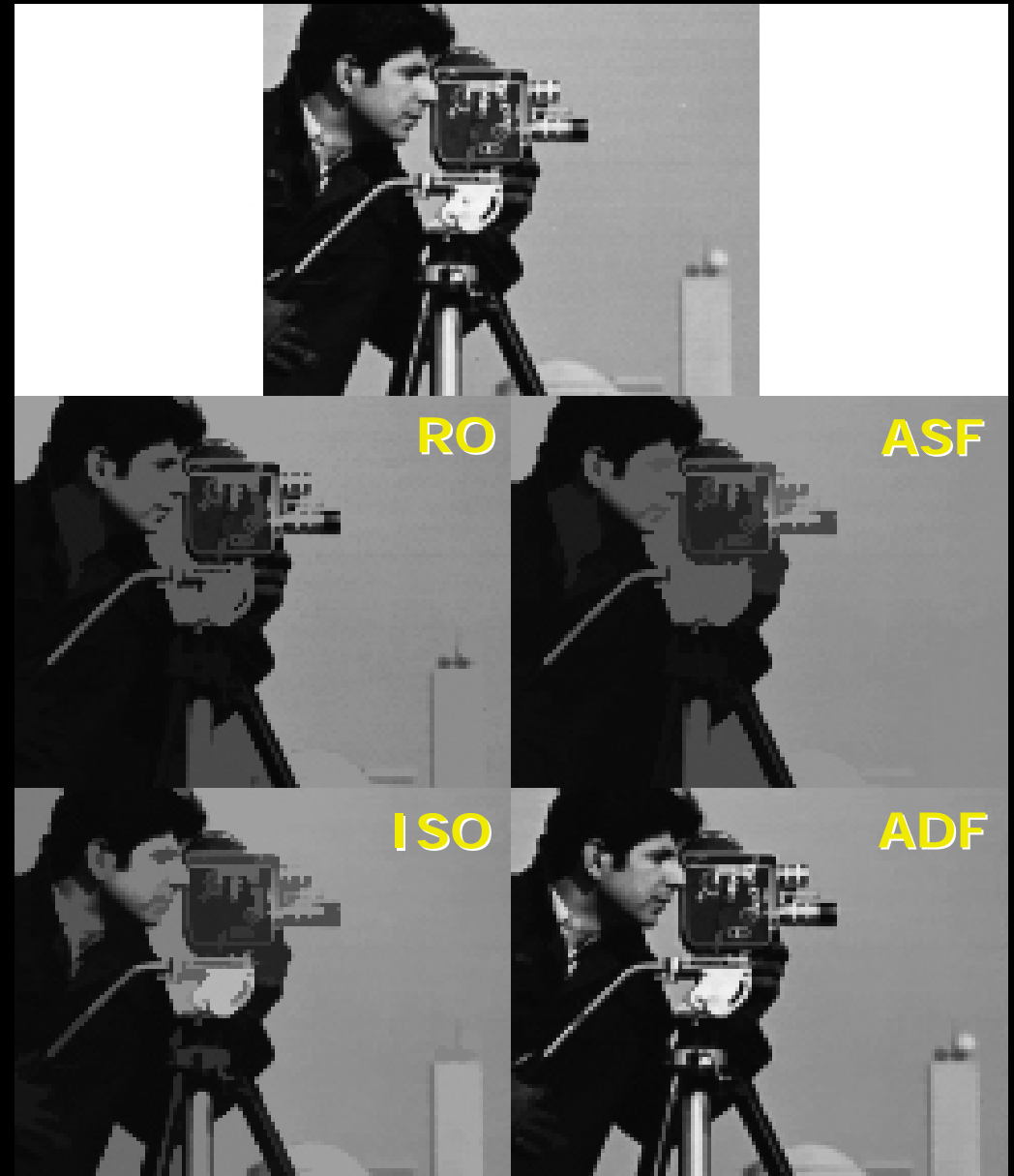
# Evaluation

*"Comparing morphological levelings constrained by different markers"*

★ **ASF** marker suppressed regional extrema in regions of proportional size with the structure element (like the top of the two buildings, in the right center of the image).

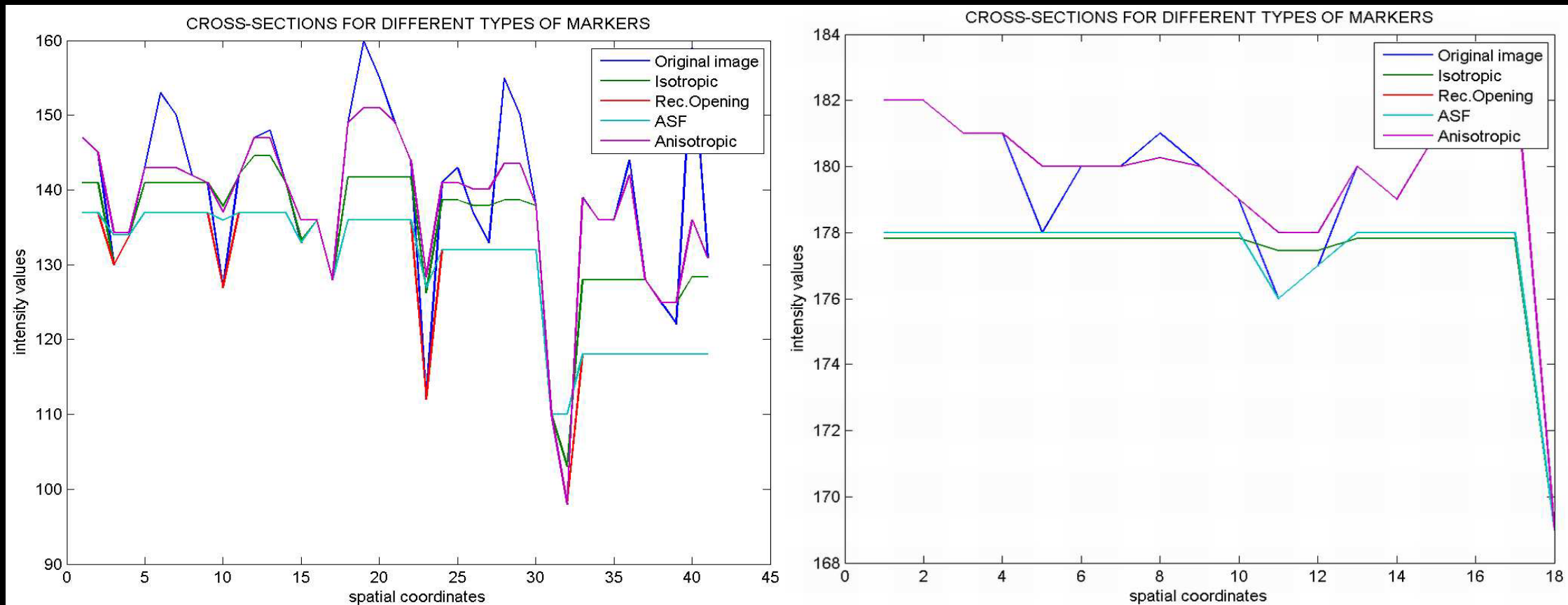
★ **RO** marker was robust in flattening bright regions with proportional size with the structure element (like the bright values in the top of the two building in the right center of the image).

★ **ADF** marker lead to a simplified version of the reference image on which the edges and the contrast have been preserved, opposite to **ISO** and the other morphological markers.



# Evaluation

*"Comparing morphological levelings constrained by different markers"*



★ **ASF** and **RO** markers resulted into the most extended simplification and drew most away from the reference image intensities.

★ **ADF** marker followed more constantly, than all the other markers, intensity values and changes between the different image zones, due to its edge preserving nature.

# Quantitative Evaluation

"Comparing morphological levelings constrained by different markers"

	Quantitative measures	Markers			
		RO	ASF	ISO	ADF
Cameraman scale 4	RMSE	13.748	15.649	10.132	4.325
	NMSE	0.011	0.014	0.006	0.001
	SSIM	0.923	0.847	0.904	0.933
Cameraman scale 7	RMSE	20.963	22.652	13.108	4.650
	NMSE	0.024	0.029	0.010	0.001
	SSIM	0.851	0.757	0.866	0.925
Crop of cameraman scale 7	RMSE	30.914	31.943	17.831	2.870
	NMSE	0.053	0.057	0.018	0.001
	SSIM	0.831	0.772	0.891	0.983

★ **ASF** and **RO** markers resulted to the broadest intensity variations.

★ **ASF** and **ISO** markers yielded to the smallest structural similarity with the original image.

★ **ADF** marker scored by far better in all quantitative measures, indicating that it preserves effectively i) changes in intensities and ii) the structural similarity with the reference image.

# Conclusions

"Comparing morphological levelings constrained by different markers"

*a framework for the comparison of levelings with different markers was developed, through the introduction of a relation between the scale parameters of all markers.*

- ✱ The **ADF** markers yielded to a simplified version of the reference image which followed most constantly image intensity (both values and changes) due to its edge preserving nature, contrary to **ISO**.
- ✱ The **ASF** and **RO** markers resulted into the most extended simplifications and differed most from the reference image intensities.

# Conclusions - Future work

"Comparing morphological levelings constrained by different markers"

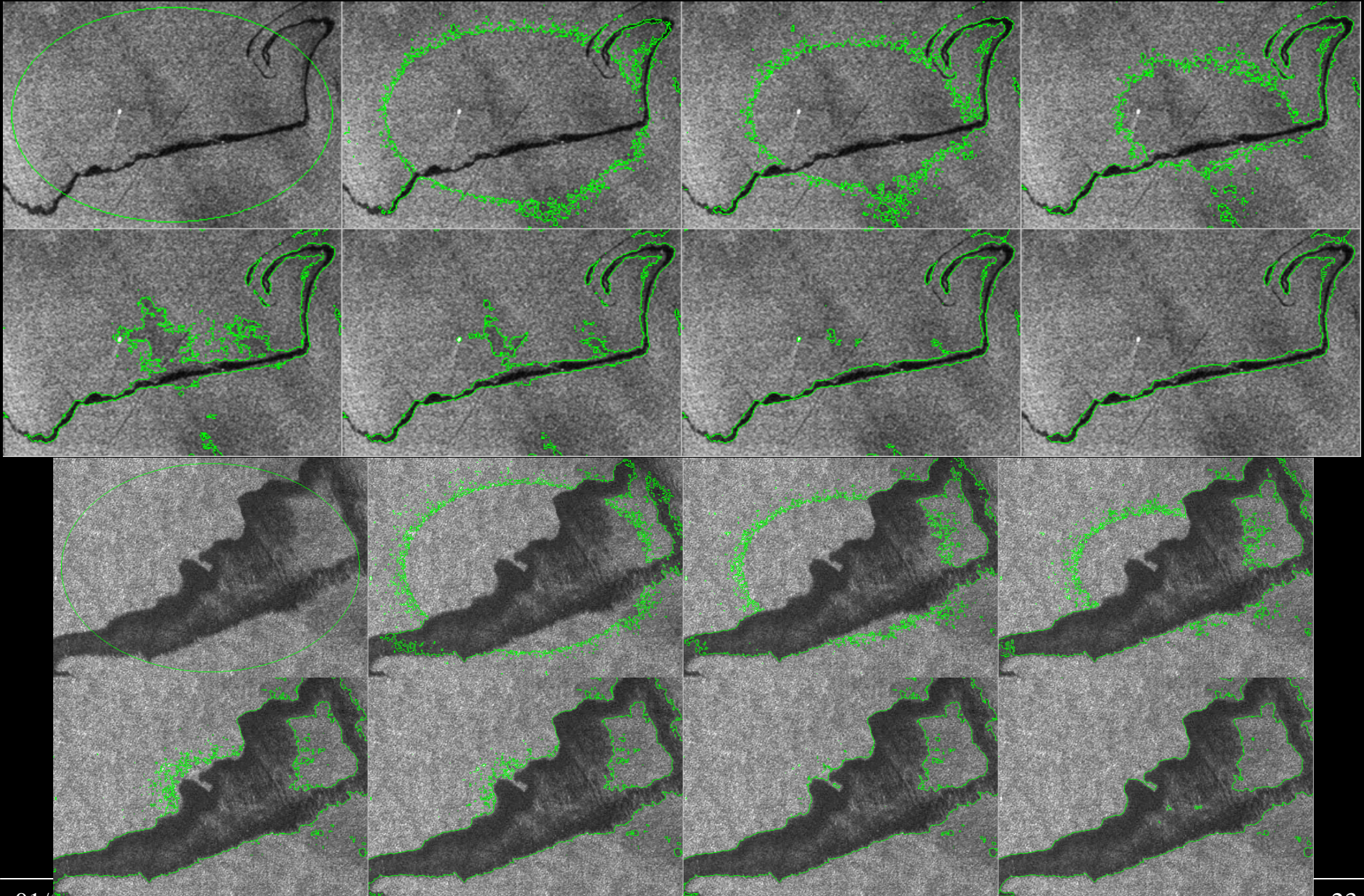
✱ The **ADF** scored by far better in all the quantitative measures (small extent of intensity variations and high structural similarity with the reference image).

✱ *Subjects for further research are*

- ✱ *the establishment of an axiomatic relation between the scales of different markers and*
- ✱ *their evaluation for specific computer vision tasks like image segmentation and the extraction of specific objects.*



# Results



# Results

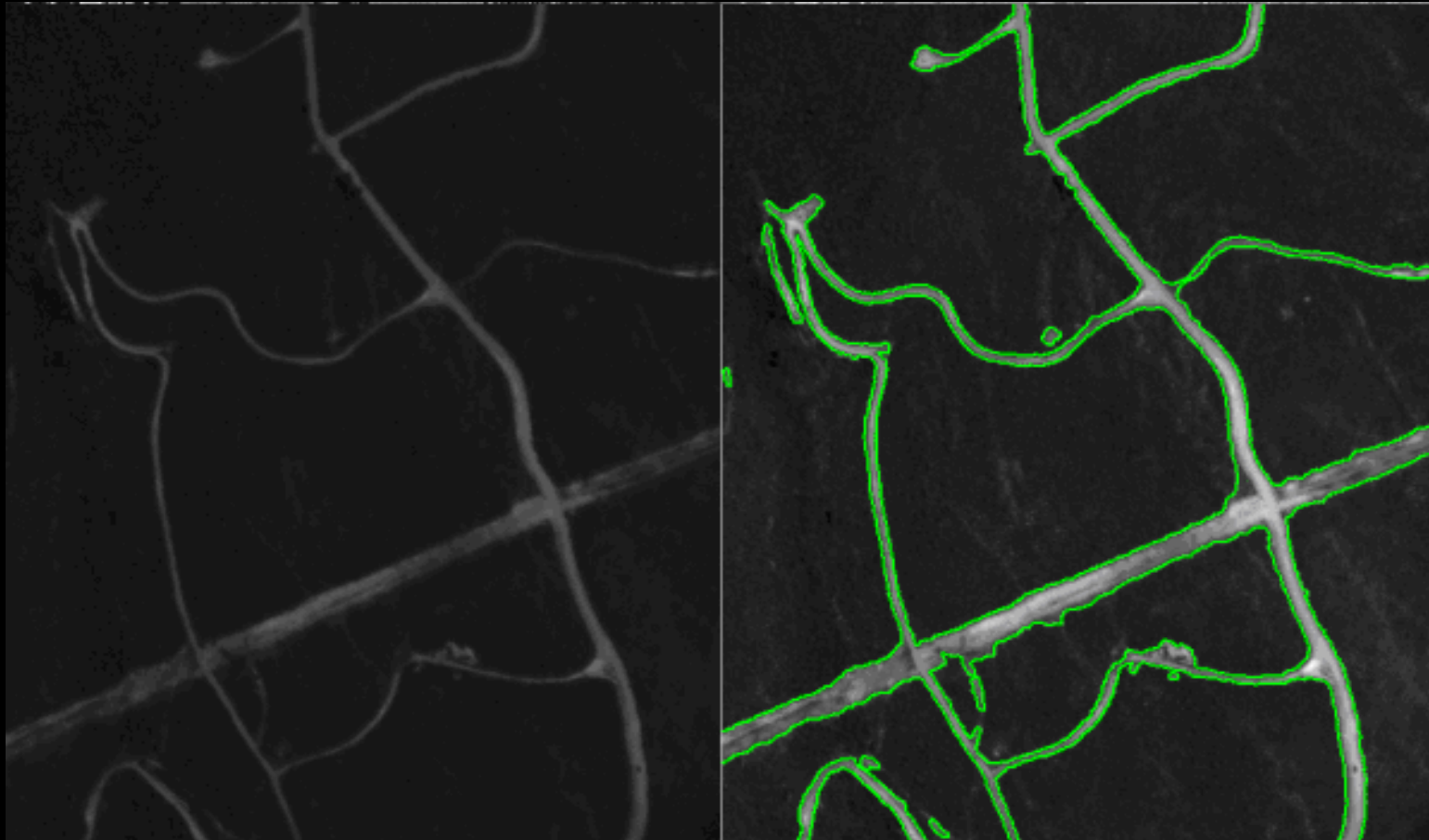




# Results

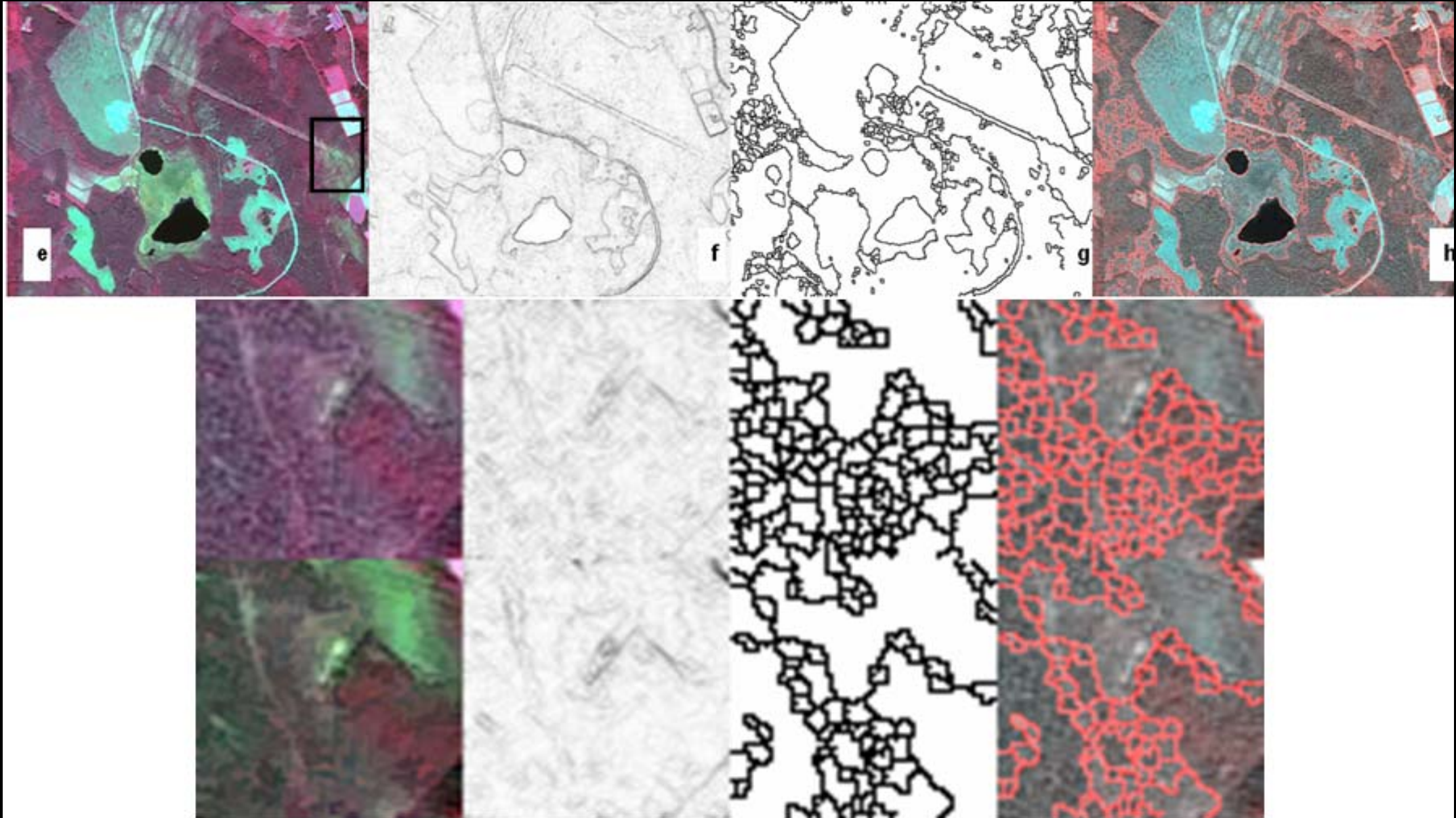


# Results



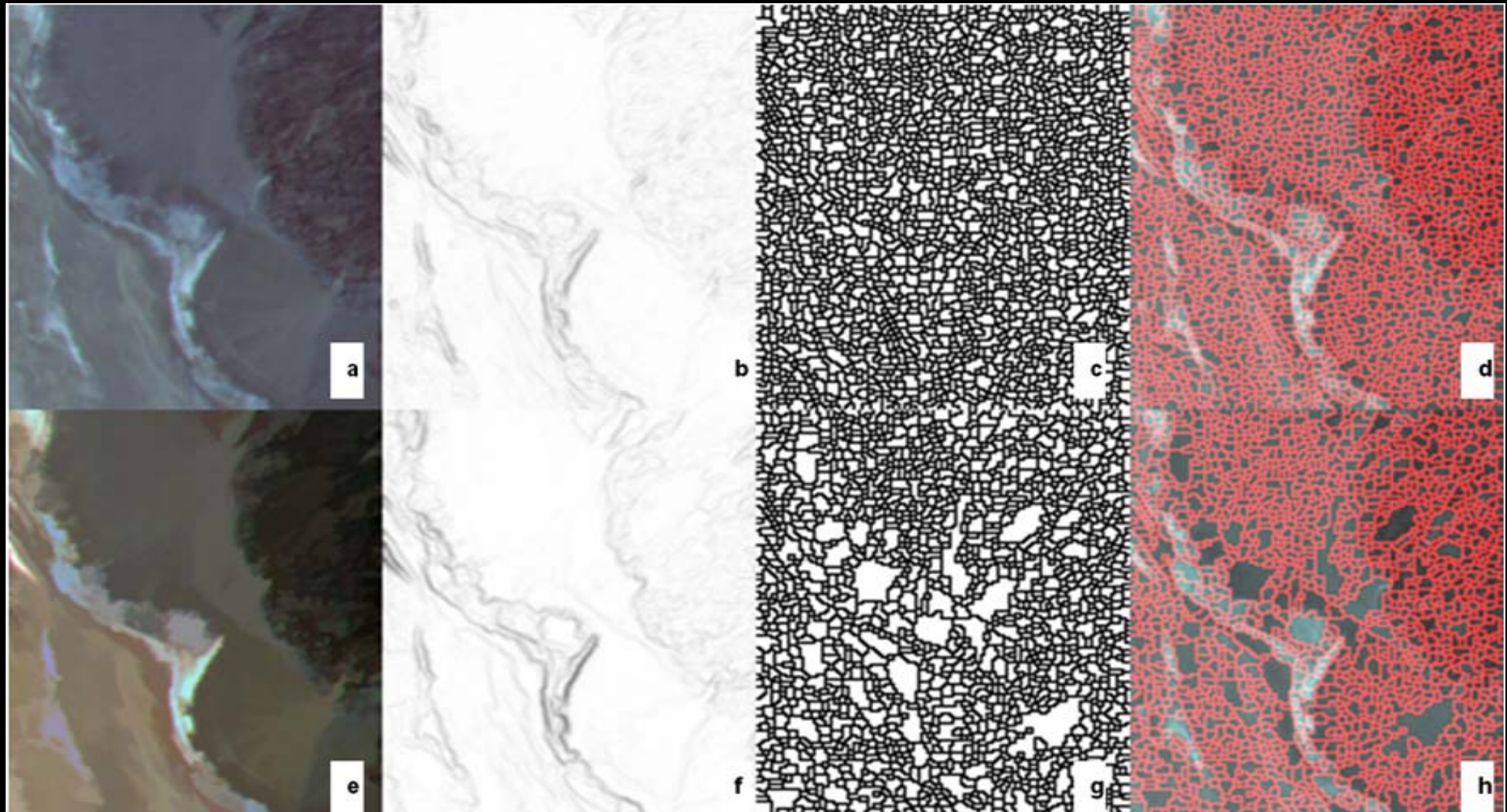


# Results





# Results



*Obrigado*

*thanks for your attention*

