On distances, paths and connections for hyperspectral image segmentation

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Contents

Hyperspectral image

Definition

Hyperspectral image: at each point x_i is associated a vector with values in spectrum, time, wavelength, or associated to an index j.

$$\mathbf{f}_\lambda : \begin{cases} E & \rightarrow & \mathcal{T}^L \quad \text{with } E \subset \mathbb{R}^2, \mathcal{T} \subset \mathbb{R} \\ x & \rightarrow & \mathbf{f}_\lambda(x) = (f_{\lambda_1}(x), f_{\lambda_2}(x), \ldots, f_{\lambda_L}(x)) \end{cases}$$

$f_{\lambda_j} \setminus j \in \{1, 2, \ldots, L\}$ is a channel (L is the number of channels)

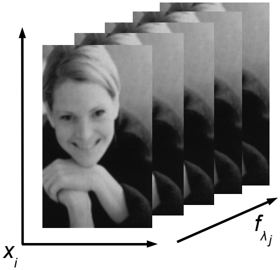


Image source: Spectral Database, University of Joensuu Color Group, `http://spectral.joensuu.fi/`

Spectral image: 400 nm to 700 nm 5 nm

Image size : $45 \times 76 \times 61$

Hyperspectral image

Image woman face in wavelength $(152 \times 91 \times 61)$

channels $\lambda = [400 : 5 : 700]$ nm

Quasi-flat zones

Definition: Quasi-flat zones or λ-flat zones (λ-FZ)

Given a distance $d : \mathcal{T}^L \times \mathcal{T}^L \to \mathbb{R}^+$, two points x, $y \in E$ belongs to the same quasi-flat zone of an hyperspectral image $\mathbf{f}_\lambda$ if and only if there is a path $(p_0, p_1, \ldots, p_n) \in E^n$ such as $p_0 = x$ and $p_n = y$ and, if, for all i, $(p_i, p_{i+1}) \in E^2$ are neighbours and $d\left(\mathbf{f}_\lambda(p_i), \mathbf{f}_\lambda(p_{i+1})\right) \leq \lambda$, with $\lambda \in \mathbb{R}^+$.

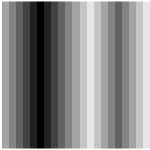
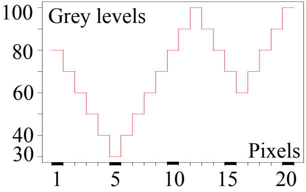
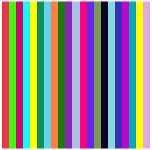


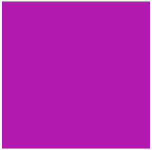
Image
(21 $\times$ 21)



Row profile



λ-FZ
$\lambda = 9{,}9$
21 zones



λ-FZ
$\lambda = 10$
1 zone

Issue

Limitations of λ-Flat Zones: they are very sensitive to parameter λ because only local information is taken into account.

Aim of the study: To solve this effect we introduce finer partitions of each λ-FZ using regional information.

Starting with an initial partition by λ-FZ with a non critical high value of λ that leads to a sub-segmentation, a second segmentation is performed based on two new connections:

- η-Bounded Regions: η-BR
- μ-Geodesic Balls: μ-GB

The corresponding algorithms are founded on seed-based region growing inside the λ-FZ.

Definitions

Definition (Partition)

Let E be an arbitrary set. A partition $\mathcal{D}$ of E is a mapping $x \rightarrow D(x)$ from E into $\mathcal{P}(E)$ such that: (i) for all $x \in E$: $x \in D(x)$, (ii) for all $x, y \in E$: $D(x) = D(y)$ or $D(x) \cap D(y) = \emptyset$. $D(x)$ is called the class of the partition of origin x.

The set of partitions of an arbitrary set E is ordered as follows.

Definition (Order of partitions)

A partition $\mathcal{A}$ is said to be finer (resp. coarser) than a partition $\mathcal{B}$, $\mathcal{A} \leq \mathcal{B}$ (resp. $\mathcal{A} \geq \mathcal{B}$), when each class of $\mathcal{A}$ is included in a class of $\mathcal{B}$.

This leads to the notion of ordered hierarchy of partitions $\Pi_{i=1}^{N} \mathcal{D}_i$, such that $\mathcal{D}_i \leq \mathcal{D}_{i+1}$, and even to a complete lattice [Serra(2006)].

Definitions

To solve this effect (too high sensitivity of λ parameter), we create finer partitions of each λ-FZ. These partitions are defined with connections.

Definition (Connection)

Let E be an arbitrary non empty set. We call connected class or connection $\mathcal{C}$ any family in $\mathcal{P}(E)$ such that: (0) $\emptyset \in \mathcal{C}$, (i) for all $x \in E$, $\{x\} \in \mathcal{C}$, (ii) for each family $C_i, i \in I$ in $\mathcal{C}$, $\cap_i C_i \neq \emptyset$ implies $\cup_i C_i \in \mathcal{C}$. Any set C of a connected class $\mathcal{C}$ is said to be connected.

Vectorial median

Definition (Vectorial median)

A vectorial median of a set $R \subset E$ is any value $\mathbf{f}_\lambda(k)$ in the set at point $k \in R$ such as:

$$k = argmin_{p \in R} \sum_{i / x_i \in R} d\left(\mathbf{f}_\lambda(p), \mathbf{f}_\lambda(x_i)\right) = argmin_{p \in R} \delta_R(\mathbf{f}_\lambda(p)) \quad (1)$$

i.e. "one of the points which minimize the sum of distances to the others".

δ_R: ascending ordered list based on the cumulative distance (of each point of R to the others).

The first element of the list δ_R is the vectorial median (the last element is considered as the anti-median).

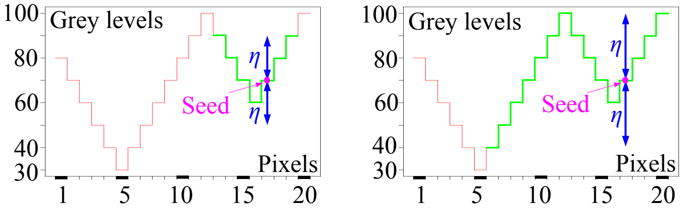
η-bounded regions η-BR

Comparison with λ-FZ

For λ-FZ, a hiker starting from one point only deals with the local slope and not with the difference in altitude on the λ-flat zones.

Principle for η-BR

Considering the difference in altitude, a hiker starting from one point x has a walk restricted to a ball of diameter $2 \times \eta$ centered on x inside a λ-FZ.



$\eta = 20$ $\eta = 30$

η-bounded regions η-BR

Definition (η-bounded connection)

Given an hyperspectral image $\mathbf{f}_\lambda(x)$ and its initial partition based on λ-flat zones, λFZ, where λFZ_i is the connected class i and $R_i \subseteq E$ (with cardinal K) is the set of points p_k, $k = 0, 1, 2, ..., K-1$, that belongs to the class i. Let p_0 be a point of R_i, named the center of class i, and let $\eta \in \mathbb{R}^+$ be a positive value. A point p_k belongs to the η-connected component centered at p_0, denoted $\eta BR_i^{\mathbf{p_0}}$, if and only if $d(\mathbf{f}_\lambda(p_0), \mathbf{f}_\lambda(p_k)) \leq \eta$ and p_0 and p_k are connected.

Seed (center of the class): 1^{st} non assigned-point of ascending ordered list based on the cumulative distance δ_R in a λ-FZ.

η-bounded regions η-BR

Construction of η-BR

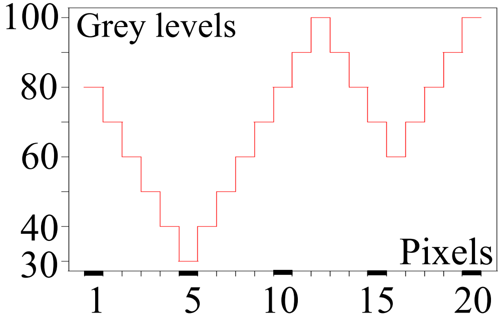
For each class λFZ_i the method is iterated with different centers p_j ($j = 0, 1, \cdots J$) until: $\cup_{j=0}^{J} \eta BR_i^{p_j} = \lambda FZ_i$, $\cap_{j=0}^{J} \eta BR_i^{p_j} = \emptyset$ where the η-bounded regions are connected.

Properties of η-BR

- Each seed p_j belongs to $\lambda FZ_i \setminus \cup_{l=0}^{j-1} \eta BR_i^{p_l}$.
- $\forall x \in E$, $\eta BR(x) \leq \lambda FZ(x)$

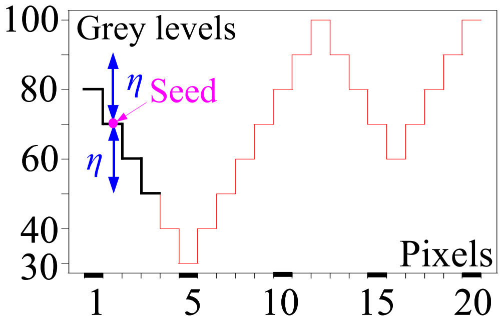
η-bounded regions η-BR

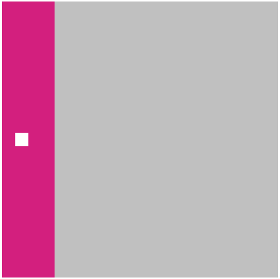
$$\lambda = 10, \quad \eta = 20$$



η-bounded regions η-BR

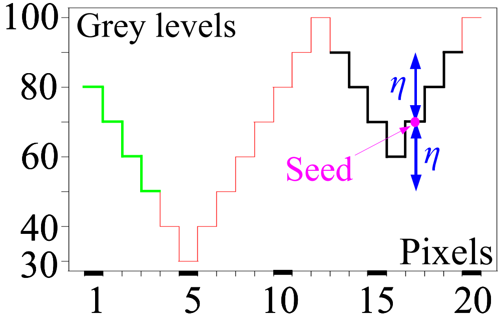
$$\lambda = 10, \quad \eta = 20$$

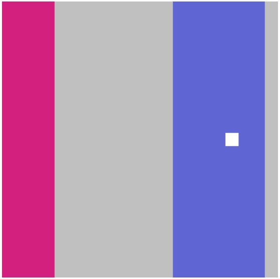




η-bounded regions η-BR

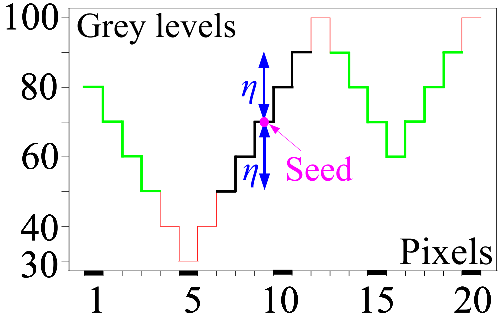
$$\lambda = 10, \quad \eta = 20$$

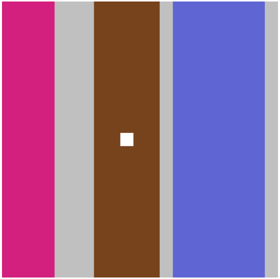




η-bounded regions η-BR

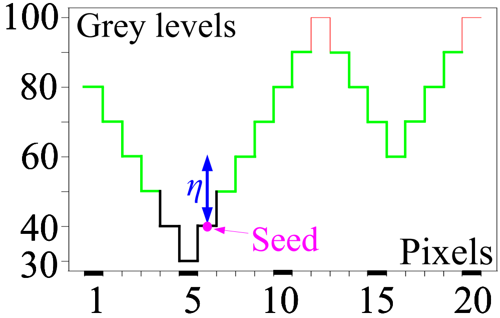
$$\lambda = 10, \quad \eta = 20$$

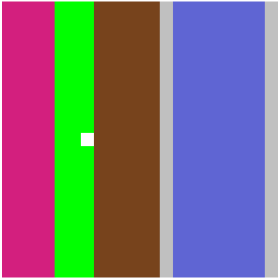




η-bounded regions η-BR

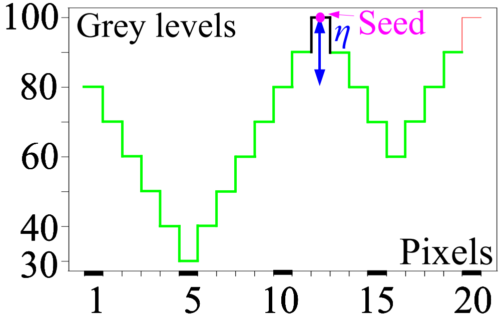
$$\lambda = 10, \quad \eta = 20$$

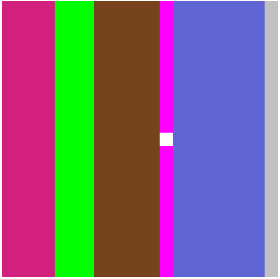




η-bounded regions η-BR

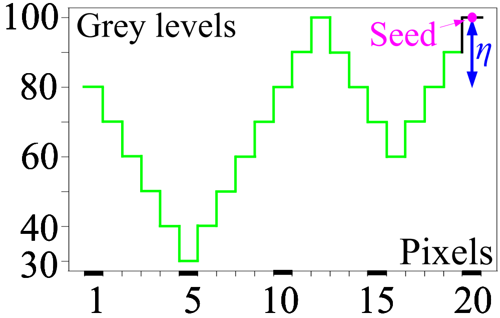
$\lambda = 10, \quad \eta = 20$

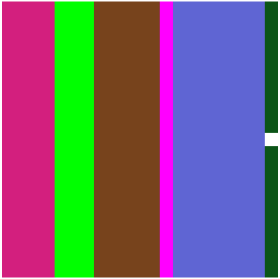




η-bounded regions η-BR

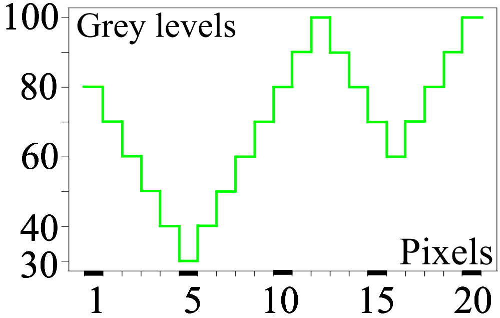
$$\lambda = 10, \quad \eta = 20$$

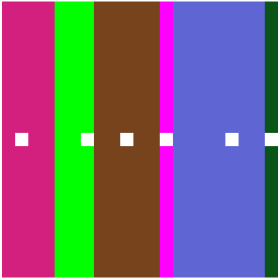




η-bounded regions η-BR

$$\lambda = 10, \quad \eta = 20$$

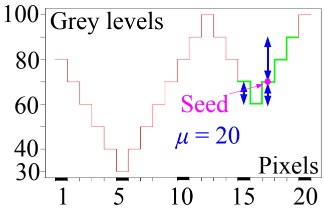




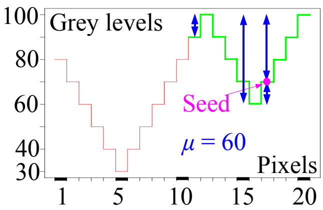
μ-geodesic balls μ-GB

Principle of μ-GB

A hiker starting from one point k has a walk restricted by a cumulative difference in altitude less than μ, inside a λ-FZ.



$\mu = 20$



$\mu = 60$

μ-geodesic balls μ-GB

Definition (μ-geodesic connection)

Given an hyperspectral image $\mathbf{f}_\lambda(x)$ and its initial partition based on λ-flat zones, λFZ, where λFZ_i is the connected class i and $R_i \subseteq E$ (with cardinal K) is the set of points p_k, $k = 0, 1, 2, \ldots, K-1$, that belongs to the class i. Let p_0 be a point of R_i, named the center of class i, and let $\mu \in \mathbb{R}^+$ be a positive value. A point p_k belongs to the μ-connected component centered at p_0, denoted $\mu GB_i^{p_0}$ if and only if $d_{geo}(\mathbf{f}_\lambda(p_0), \mathbf{f}_\lambda(p_k)) \leq \mu$.

Seed (center of the class): 1^{st} non assigned-point of ascending ordered list based on the cumulative distance δ_R in a λ-FZ.

μ-geodesic balls μ-GB

Construction of μ-GB

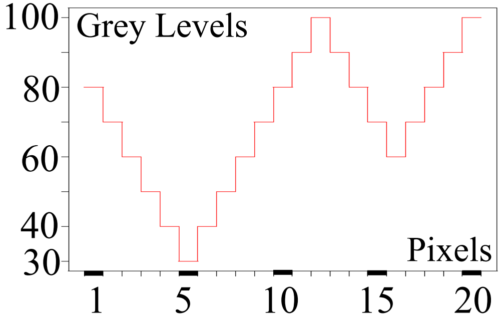
μ-geodesic balls are built as η-bounded regions, except that from each seed the geodesic ball is computed inside the λ-FZ.

Properties of μ-GB

- Each seed p_j belongs to $\lambda FZ_i \setminus \cup_{l=0}^{j-1} \mu GB_i^{p_l}$.
- $\forall x \in E$, $\mu GB(x) \leq \lambda FZ(x)$
- we have a regional control of the "geodesic size" of the classes

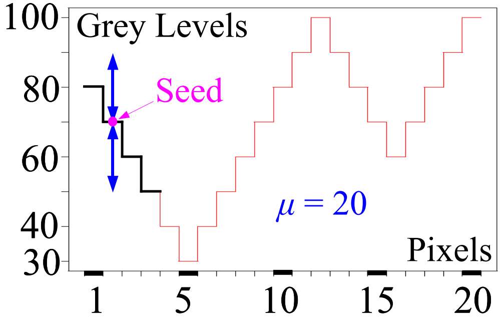
μ-geodesic balls μ-GB

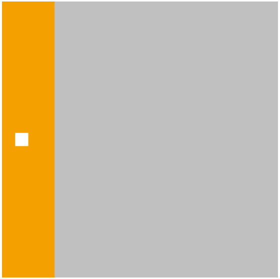
$$\lambda = 10, \quad \mu = 20$$



μ-geodesic balls μ-GB

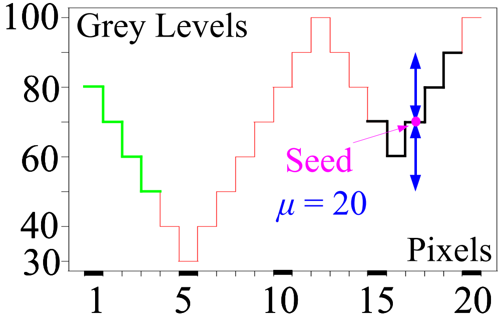
$$\lambda = 10, \quad \mu = 20$$

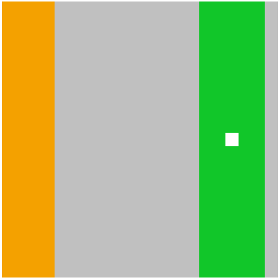




μ-geodesic balls μ-GB

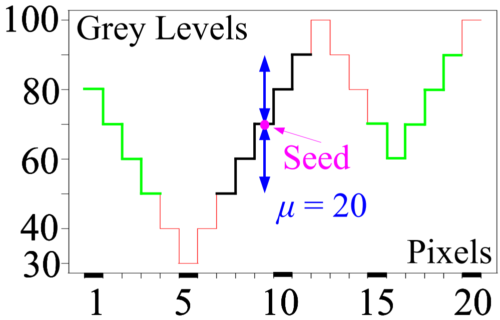
$$\lambda = 10, \quad \mu = 20$$

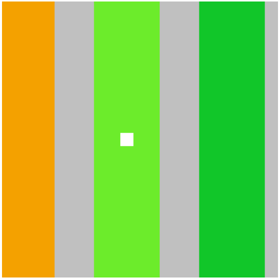




μ-geodesic balls μ-GB

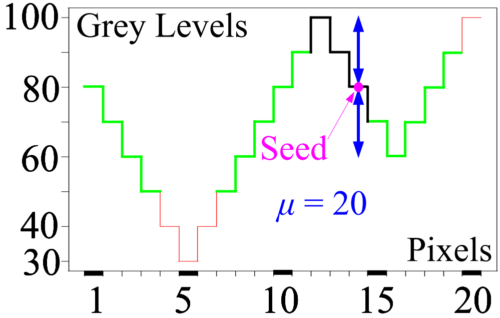
$$\lambda = 10, \quad \mu = 20$$

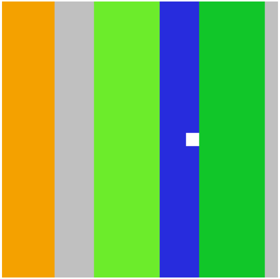




μ-geodesic balls μ-GB

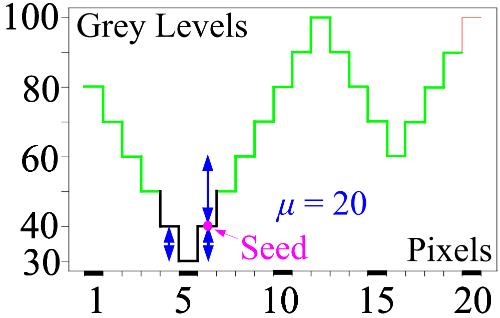
$$\lambda = 10, \quad \mu = 20$$

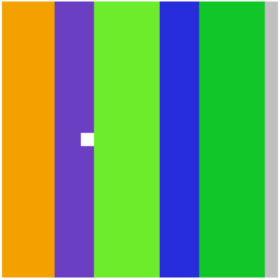




μ-geodesic balls μ-GB

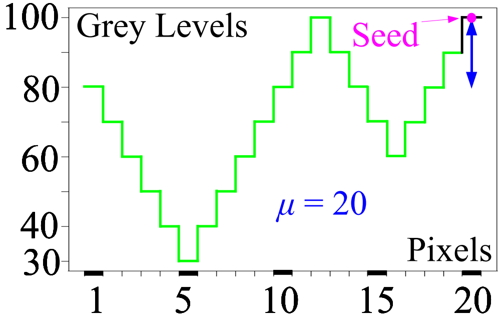
$$\lambda = 10, \quad \mu = 20$$

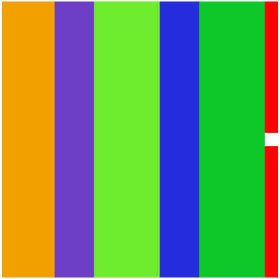




μ-geodesic balls μ-GB

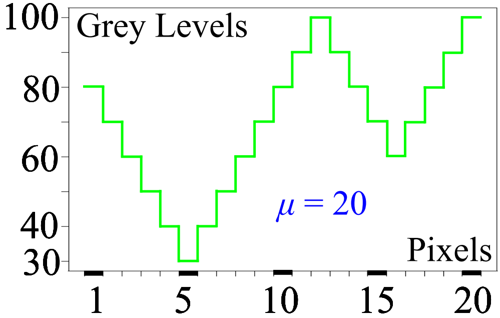
$$\lambda = 10, \quad \mu = 20$$



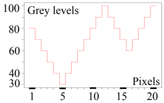


μ-geodesic balls μ-GB

$$\lambda = 10, \quad \mu = 20$$



Comparison between η-BR and μ-GB

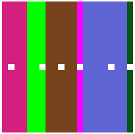


Row profile

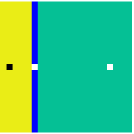
λ-FZ
$\lambda = 10$

η-BR
$\eta = 0$

η-BR
$\eta = 10$

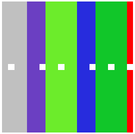


η-BR
$\eta = 20$

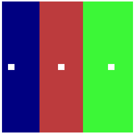


η-BR
$\eta = 30$

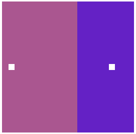
μ-GB
$\mu = 0$



μ-GB
$\mu = 20$

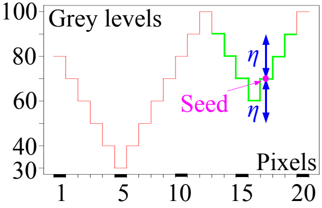


μ-GB
$\mu = 40$

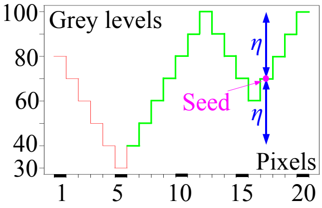


μ-GB
$\mu = 100$

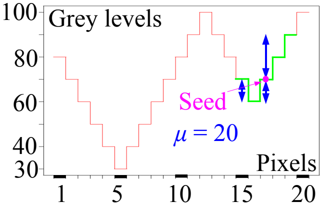
Comparison between η-BR and μ-GB



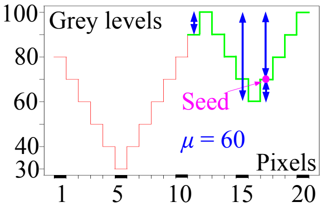
η-BR $\eta = 20$



η-BR $\eta = 30$



μ-GB $\mu = 20$



μ-GB $\mu = 60$

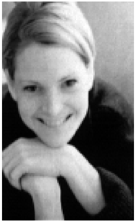
Comparison between η-BR and μ-GB

	η-BR	μ-GB
Sensitivity to bumps (and hollows)	$+$	$-$
Control of the size of the region	$-$	$+$

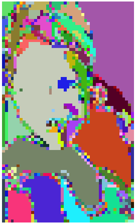
Results in image space

Generalization to hyperspectral images:

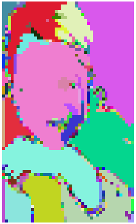
- use of vector distances (Chi-Squared, Euclidean)
- seed: vectorial median and not the minima or the maxima which does not exist.



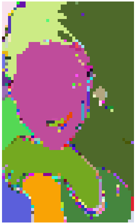
$f_{\lambda_{61}}$
$(45 \times 76 \times 61)$



λ-FZ
$\lambda = 0.003$



λ-FZ
$\lambda = 0.004$

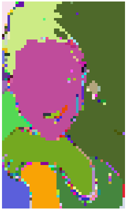


λ-FZ
$\lambda = 0.005$

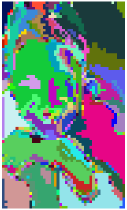
λ-FZ
$\lambda = 0.006$

Chi-squared distance is used d_{χ^2}.

Results in image space



λ-FZ
$\lambda = 0.005$



η-BR
$\eta = 0.007$

η-BR
$\eta = 0.009$

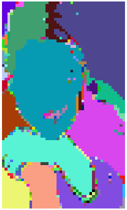
η-BR
$\eta = 0.011$

η-BR
$\eta = 0.02$

μ-GB
$\mu = 0.01$

μ-GB
$\mu = 0.02$

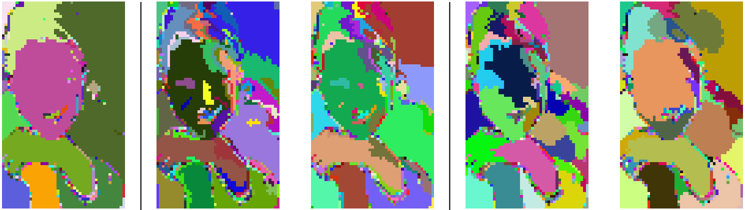
μ-GB
$\mu = 0.03$



μ-GB
$\mu = 0.05$

Results in image space

Seed: anti-median. Few changes are observed.
$\Longrightarrow$ robustness of the method to the seeds choice.



λ-FZ	η-BR	η-BR	μ-GB	μ-GB
$\lambda = 0.005$	$\eta = 0.009$	$\eta = 0.011$	$\mu = 0.02$	$\mu = 0.03$

Chi-squared distance is used d_{χ^2}.

Results in factor space

FCA (Factor Correspondence Analysis) [Benzécri(1973)]

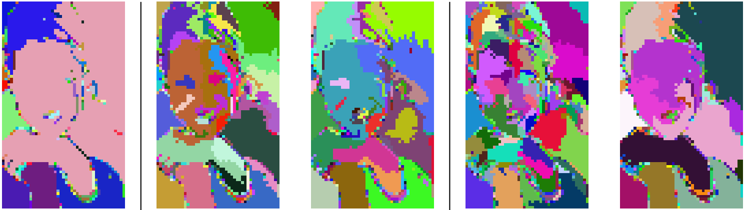
Factor axes filtered by leveling (structuring element squared 3×3).

$c^{\mathbf{f}}_{\alpha_1}$
inertia 87.8%

$c^{\mathbf{f}}_{\alpha_2}$
inertia 10.2%

$c^{\mathbf{f}}_{\alpha_3}$
inertia 1.5%

Results in factor space



λ-FZ	η-BR	η-BR	μ-GB	μ-GB
$\lambda = 0.04$	$\eta = 0.1$	$\eta = 0.15$	$\mu = 0.1$	$\mu = 0.3$

Euclidean distance is used d_E.

Some other results on a satellite image

f_{λ_1} blue

©CNES

f_{λ_2} green

©CNES

f_{λ_3} red

©CNES

f_{λ_4} proche IR

©CNES

f_{λ_5} panchrom.

©CNES

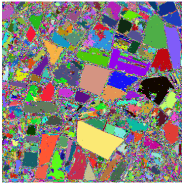
synthetic RGB

Image "Roujan" $365 \times 365 \times 5$ pixels. Resolution 0.70 meters.
Source: CNES (French space agency) + Pr. G. Flouzat (Laboratoire de Télédétection à Haute Résolution, Toulouse 3)

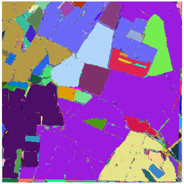
Some other results on a satellite image

Image space: Chi-squared distance is used d_{χ^2}.

synthetic RGB



λ-FZ
$\lambda = 0.005$

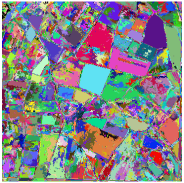


λ-FZ
$\lambda = 0.015$

Some other results on a satellite image

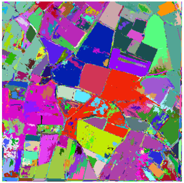
Image space: Chi-squared distance is used d_{χ^2}.

synthetic RGB

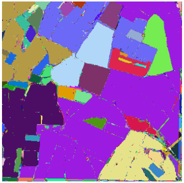


η-BR
$\eta = 0.02$

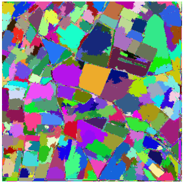
η-BR
$\eta = 0.03$



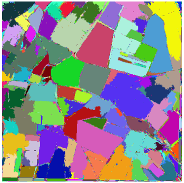
η-BR
$\eta = 0.04$



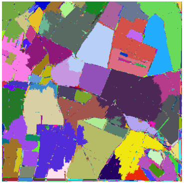
λ-FZ
$\lambda = 0.015$



μ-GB
$\mu = 0.1$



μ-GB
$\mu = 0.2$



μ-GB
$\mu = 0.3$

Application fields

- Definition of homogeneous regions useful as markers for watershed segmentation
- Detection and characterization of textured regions

Conclusions and perspectives

- η-BR and μ-GB improve λ-FZ (local information) by introduction of regional information.
- η-bounded connection and μ-geodesic connection are connections of 2^{nd} order because they are included in λ-FZ.
- The approach consists in selecting a sufficiently high parameter λ to obtain first a sub-segmentation.
- These connections lead to a pyramid of partition which is not an ordered pyramid.
- They are the generalization of the jump connection [Serra(1999)] with the difference that the seeds p_j cannot be the minima or maxima. Seed: vector median.
- Perspective: to select locally for each λ-FZ the value for η or μ.

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