Stochastic watershed segmentation

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Plan

Introduction

Motivation

- watershed-based algorithms for image segmentation: markers, non-parametric pyramids, hierarchies according to morphological criteria, interactive segmentation
- in many applications from natural images: unsupervised segmentation in very few regions

Example: Volumic extinction values to determine the R most significant regions

$\mathbf{f}$

$\varrho^{LS+H}(\mathbf{f})$

$sg^{R-vol}(\mathbf{f}, 10)$

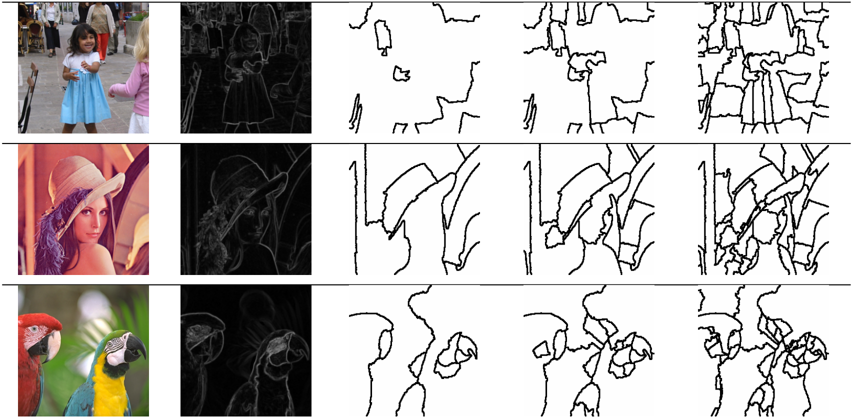
$sg^{R-vol}(\mathbf{f}, 20)$

$sg^{R-vol}(\mathbf{f}, 50)$

Introduction

More examples:

$\mathbf{f}$ | $\varrho^{LS+H}(\mathbf{f})$ | $sg^{R-vol}(\mathbf{f}, 10)$ | $sg^{R-vol}(\mathbf{f}, 20)$ | $sg^{R-vol}(\mathbf{f}, 50)$



Introduction

Aim

- to improve the watershed-based segmentation in very few regions ($R \sim 10, 20$)
- to detect the most significant contours from the gradient: the contours which are robust with respect to variations in the segmentation conditions
- to work in a probabilistic framework in order to build a pdf of contours by Monte-Carlo simulations
- to use the pdf of contours for defining the most significant regions

Main concept

- The classical paradigm of watershed segmentation lays on the appropriate choice of markers: the most intelligent part resides in the development of criteria used to select the required markers
- Stochastic paradigm: To select random germs for markers. This arbitrary choice is balanced by the use of a given number of realizations

Transformations and main operators (1/2)

Colour gradient $\varrho^{LS+H}(\mathbf{f}(\mathbf{x})) =$
$$f_S(x) \times \varrho^\circ(f_H(x)) + (1 - f_S(x)) \times \varrho(f_L(x)) + \varrho(f_S(x))$$

Watershed segmentation

- Marker-based segmentation (image of markers $mrk(x)$):
  $sg^{mrk}(\mathbf{f}(x)) = Wshed(\varrho^{LS+H}(\mathbf{f}(x)), mrk(x))$
- Volumic-based segmentation in R regions:
  $sg^{R-vol}(\mathbf{f}(x)) = Wshed(\varrho^{LS+H}(\mathbf{f}), R)$

Markers: Random points

- N Spatially uniform points: $mrk_i(x) = uniform(N)$
- Regionalized Poisson points of density $\theta(x)$:
  $mrk_i^\theta(x) = poisson(\theta)$
- M: Number of realizations ($i = 1, 2, \cdots, M$)

Transformations and main operators (2/2)

Probability density functions of contours (Parzen estimate):

$$pdf(x) = \frac{1}{M}\sum_{i=1}^{M} k_i(x) = \frac{1}{M}\sum_{i=1}^{M} sg_i^{mrk}(\mathbf{f}(x)) * G_\sigma$$

G_σ is a gaussian filter,
$sg_i^{mrk}(\mathbf{f}(x)) = Wshed(\varrho^{LS+H}(x), mrk_i(x))$

Probabilistic segmentation from wshed on pdf In R volumic regions:

$$sg^{R-vol}(pdf(x)) = Wshed(pdf(x), R)$$

Probabilistic segmentation from wshed on probabilistic gradient

$$\rho(x) = \omega_1 \varrho^{LS+H}(\mathbf{f}(x)) + \omega_2 pdf(x); \;\; \omega_1 = (1-\lambda),\, \omega_1 = \lambda$$

$$sg^{R-vol}(\rho(x)) = Wshed(\rho(x), R)$$

Remarks

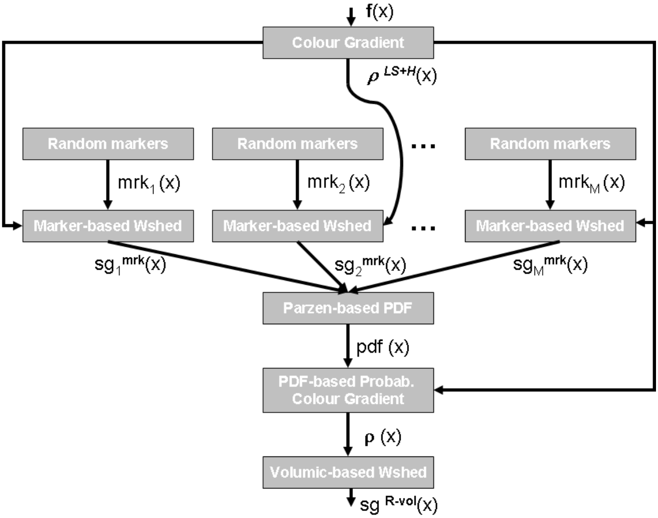
Interpretation

- Each catchment basin (each minima) of $pdf(x)$ corresponds to one the regions of the sum (or union) of different $sg_i^{mrk}(x)$.
- The integral of each catchment basin corresponds to the probability to be region of the segmentation.
- The volumic watershed of $pdf(x)$ yields the regions according to their probabilities.

Implementation

- The M realizations of contours segmentation are obtained from the same function (i.e., colour gradient) using different markers.
- Consequently, working on the neighbourhood graph and its MST, the R random markers can be considered as N random nodes of the MST.

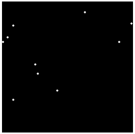
Algorithm


f(x)
Colour Gradient
ρ LS+H(x)
Random markers
Random markers
...
Random markers
mrk1 (x)
mrk2 (x)
mrkM (x)
Marker-based Wshed
Marker-based Wshed
...
Marker-based Wshed
sg1mrk(x)
sg2mrk(x)
sgMmrk(x)
Parzen-based PDF
pdf (x)
PDF-based Probab. Colour Gradient
ρ (x)
Volumic-based Wshed
sg R-vol(x)

Uniform random markers and watershed on colour gradient

$i = 1$

$mrk_i(x)$
$N = 10$

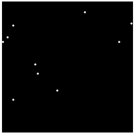


$sg_i^{mrk}(x)$

Uniform random markers and watershed on colour gradient

$i = 1$

$mrk_i(x)$
$N = 10$



$sg_i^{mrk}(x)$

Uniform random markers and watershed on colour gradient

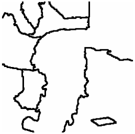
$i = 1$ | $i = 2$ | $i = 3$ | $i = 4$

$mrk_i(x)$
$N = 10$

$sg_i^{mrk}(x)$

pdf estimate and volumic watershed

$R = 10$

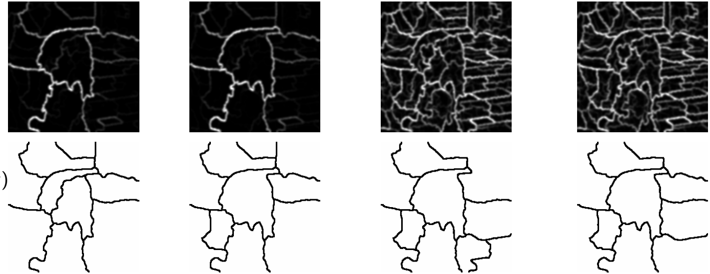


$N = 10,\ M = 50$ | $N = 10,\ M = 100$ | $N = 100,\ M = 100$ | $N = 100,\ M = 200$

$pdf(x)$

$sg^{R-vol}(x)$

$R = 10$

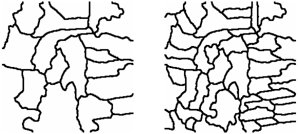


pdf estimate and volumic watershed

$R = 20$ $R = 50$

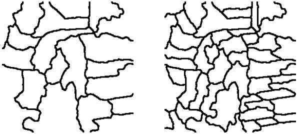
$N = 50,\ M = 100$

$R = 20$ $R = 50$



$N = 100,\ M = 200$

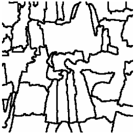
$R = 20$ $R = 50$



pdf estimate and volumic watershed

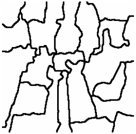
$R = 20$

$R = 50$

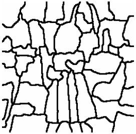


$N = 50, \; M = 100$

$R = 20$

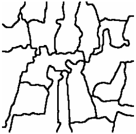


$R = 50$

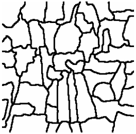


$N = 100, \; M = 200$

$R = 20$



$R = 50$



pdf estimate and volumic watershed

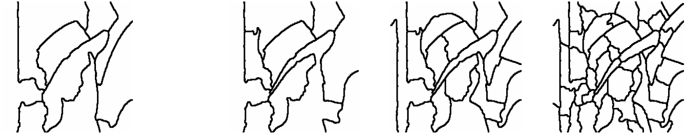
$R = 10$ $R = 20$ $R = 50$

$N = 10,\ M = 100$

$R = 10$

$N = 100,\ M = 200$

$R = 10$ $R = 20$ $R = 50$



pdf estimate and volumic watershed

$R = 10$

$R = 20$

$R = 50$

$N = 10,\ M = 100$

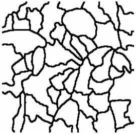
$R = 10$

$N = 100,\ M = 200$

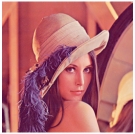
$R = 10$

$R = 20$

$R = 50$

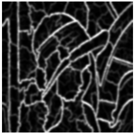


Probabilistic gradient and volumic watershed



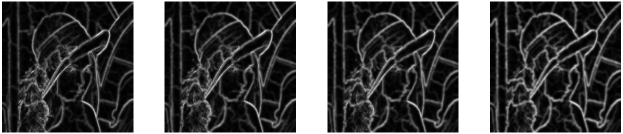
$\varrho^{LS+H}(x)$

$pdf(x)$



$\lambda = 0,33$ $\lambda = 0,43$ $\lambda = 0,50$ $\lambda = 0,66$

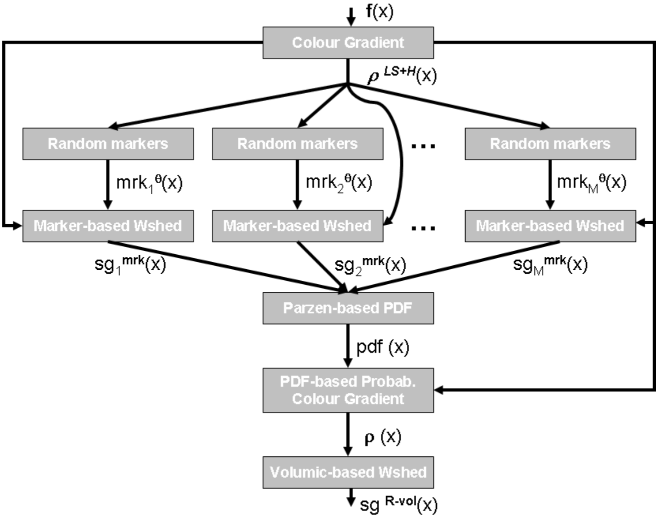
$\rho(x)$



$sg_i^{mrk}(x)$

$R = 20$

Algorithm


f(x)
Colour Gradient
ρ LS+H(x)
Random markers
Random markers
Random markers
mrk1θ(x)
mrk2θ(x)
mrkMθ(x)
Marker-based Wshed
Marker-based Wshed
Marker-based Wshed
sg1mrk(x)
sg2mrk(x)
sgMmrk(x)
Parzen-based PDF
pdf (x)
PDF-based Probab. Colour Gradient
ρ (x)
Volumic-based Wshed
sg R-vol(x)

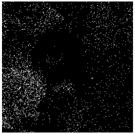
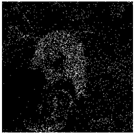
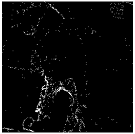
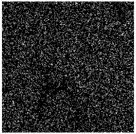
Function of regionalization to define random markers

f_L

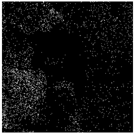
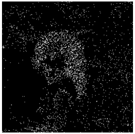
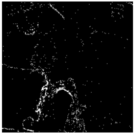
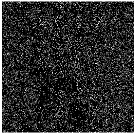
$\varrho^{LS+H}(\mathbf{f})$

$\theta = f_L$ | $\theta = \overline{f_L}$ | $\theta = \varrho^{LS+H}$ | $\theta = \overline{\varrho^{LS+H}}$

$mrk_1^{\theta}(x)$

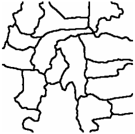
$mrk_2^{\theta}(x)$

Regionalized random markers, pdf estimate and volumic watershed

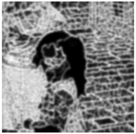
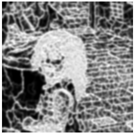
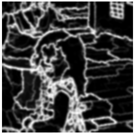
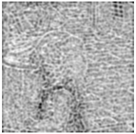
$R = 20$

$N = 100,\ M = 200,\ R = 20$



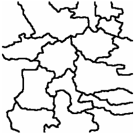
$\theta = f_L,\ M = 100$ | $\theta = \overline{f_L},\ M = 100$ | $\theta = \varrho^{LS+H},\ M = 100$ | $\theta = \overline{\varrho^{LS+H}},\ M = 100$

$pdf(x)$

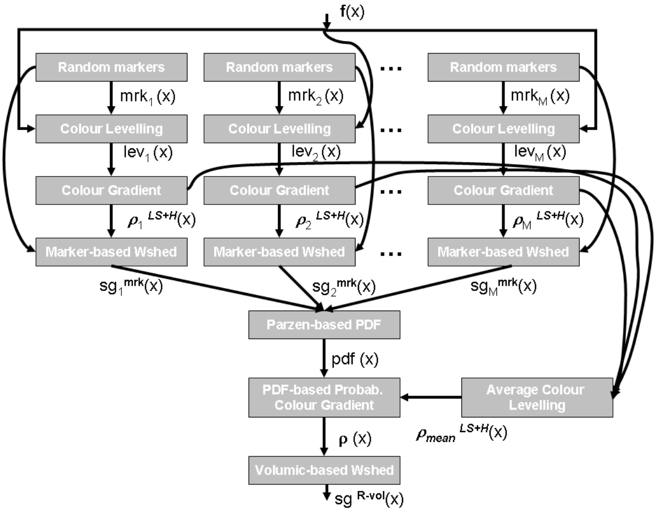





$sg^{R-vol}(x)$

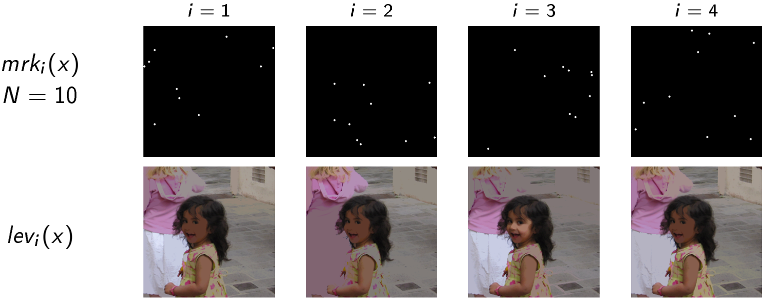
$R = 20$

Algorithm


f(x)
Random markers
Random markers
Random markers
mrk1 (x)
mrk2 (x)
mrkM (x)
Colour Levelling
Colour Levelling
Colour Levelling
lev1 (x)
lev2 (x)
levM (x)
Colour Gradient
Colour Gradient
Colour Gradient
ρ1 LS+H(x)
ρ2 LS+H(x)
ρM LS+H(x)
Marker-based Wshed
Marker-based Wshed
Marker-based Wshed
sg1 mrk(x)
sg2 mrk(x)
sgM mrk(x)
Parzen-based PDF
pdf (x)
PDF-based Probab. Colour Gradient
Average Colour Levelling
ρmean LS+H(x)
ρ (x)
Volumic-based Wshed
sg R-vol(x)

Uniform random markers, levelled colour images

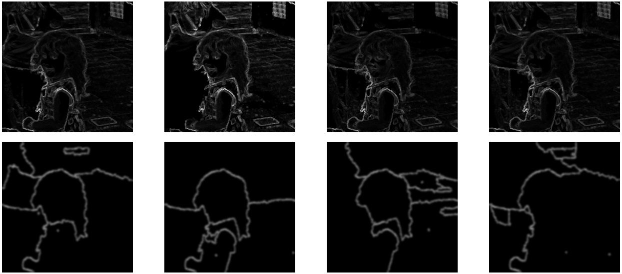


Uniform random markers, levelled-based colour gradients

$i=1$ $i=2$ $i=3$ $i=4$

$\varrho_i^{LS+H}(x)$
$N=10$

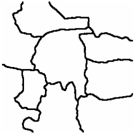
$sg_i^{mrk}(x)$



pdf estimate and volumic watershed

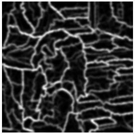
$R = 10$

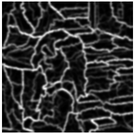
$N = 100,\ M = 200,\ R = 10$



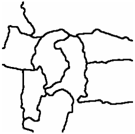
$N = 10,\ M = 50$ | $N = 10,\ M = 100$ | $N = 100,\ M = 100$ | $N = 100,\ M = 200$

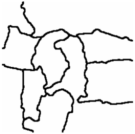
$pdf^{Lev}(x)$



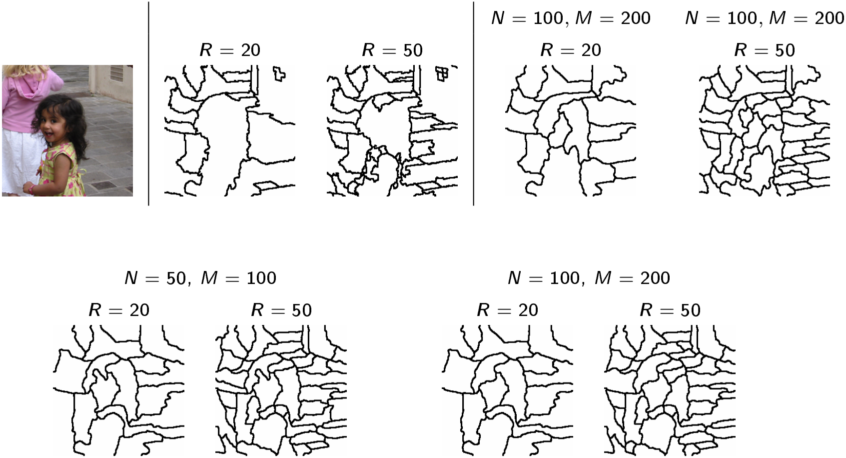


$sg^{R-vol}(x)$
$R = 10$





pdf estimate and volumic watershed



Averaged probabilistic gradient and volumic watershed

$\varrho^{LS+H}(c)$

$N = 100,\ M = 200$

$pdf^{lev}(x)$

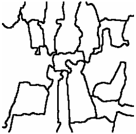
$N = 100,\ M = 200$

$\rho(x)$

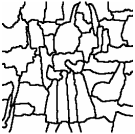
$\lambda = 0.5$

$R = 10$

$R = 20$



$R = 50$



$N = 100,\ M = 200$

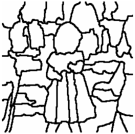
$\rho^{lev}(x)$

$\lambda = 0.5$

$R = 10$

$R = 20$

$R = 50$



Conclusions

- Complex images segmented in a few regions: both (uniform and regionalized) probabilistic approaches outperform the standard watershed segmentation.
- Images presenting specific objects on an homogeneous background: the improvement is less important.
- Uniform segmentation is mainly depending on parameter N (number of random points) which is related to R (number of regions to be determined): $N >> R$.
- Regionalized segmentation depends on parameter S (spacing for Poisson density function) and the properties of colour gradient.
- Probabilistic approaches working on the same gradient can be adapted to a graph-based framework to obtain an optimized algorithm. The approach using random marker levellings has a huge computational load (time of computation).

Perspectives

- Other variants:
  - Evolved random point simulations (structural grids, conditional models, etc.)
  - Multi-scale framework (working on image pyramids)
- To study the relation with other techniques such as random walks, Galton-Watson processus, etc.

- Probabilistic approaches combining colour gradients and texture information

Two types of contours associated to watershed

- 1st order contours: corresponding to "significant" regions and relatively independent from markers.
- 2nd order contours: corresponding to "small", "low contrasted" or "textured" regions and depend strongly on the place of markers.

It is possible to determine the type of each contour without using a probabilistic approach?