The Image Foresting Transform from the Image Domain to the Feature Space

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What is the IFT ?

The Image Foresting Transform (IFT) is a tool primarily to the design of image processing operators based on connectivity [1].

Contributors

Several colleagues and students who appear in the list of references at the end of this talk.

Outline

- A short review on the IFT.

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- How to use it in image processing with a few examples.
- Its extension to the feature space.
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- On-going works and open problems.

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  - to understand the relation among some image operators [3, 4, 5, 6], and
  - possible extensions [7, 8, 9, 10, 11, 12].

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- Efficiency: Most image operators can be implemented in linear time and further optimizations are possible with differential computation [13, 14] and for some specific applications [15, 16, 17, 18].

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- Efficiency: Most image operators can be implemented in linear time and further optimizations are possible with differential computation [13, 14] and for some specific applications [15, 16, 17, 18].
- Simplicity: The image operators are reduced to the choice of a few parameters in the IFT algorithm and a local processing of its output.

What kind of problems the IFT solve?

Problems which are either directly or indirectly related to an optimal image (set) partition.

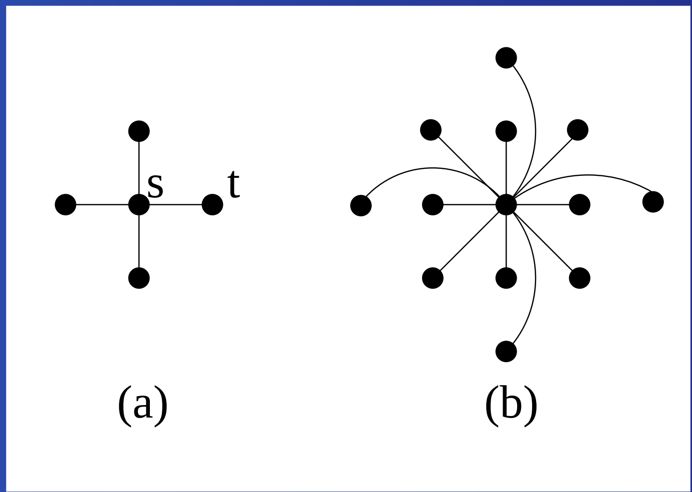
- Distance transforms and related operators: Euclidean distance transform [19], multiscale skeletonization [19], fractal dimensions [7], shape filtering [15, 19], shape saliences [7, 20, 21], shape description [20, 22], tensor scale computation [22], geodesic paths, etc.

What kind of problems the IFT solve?

- Image filtering and segmentation:
  Morphological reconstructions [3] and image segmentation based on watershed transforms [4, 6, 13, 23], live wire [16, 24], tree pruning [8, 25, 26], graph-cut measures [9], and fuzzy-connected components [10, 17].
- Pattern recognition:
  Data clustering [11] and supervised pattern classification [12, 27].

Images as graphs

The image is interpreted as a graph whose the nodes are the pixels and the arcs are defined by an adjacency relation $\mathcal{A} : (s,t) \in \mathcal{A}$ if $\|t - s\| \leq d_i$.



(a) $d_i = 1$ and (b) $d_i = 2$.

Paths in the graph

- A path π_t is a sequence of adjacent nodes with terminus at some node t.

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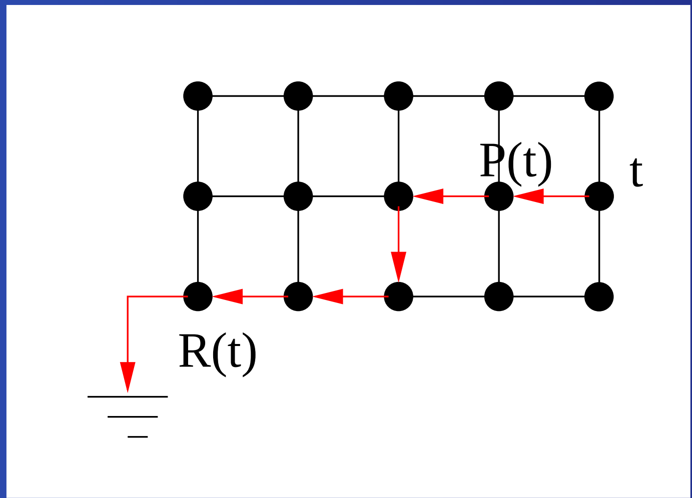
- The predecessor $P(s)$ of each node $s \in \pi_t$ leads to a root node $R(t)$ and $P(R(t)) = nil$.



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- The dual definition $f(\pi_t) \geq f(\tau_t)$ is also valid.

The IFT

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- The IFT minimizes (maximizes) a final path-value map $V(t) = \min_{\forall \pi_t}\{f(\pi_t)\}$ for every node t.
- The result is an optimum-path forest P — an acyclic graph where all paths are optimum according to f.

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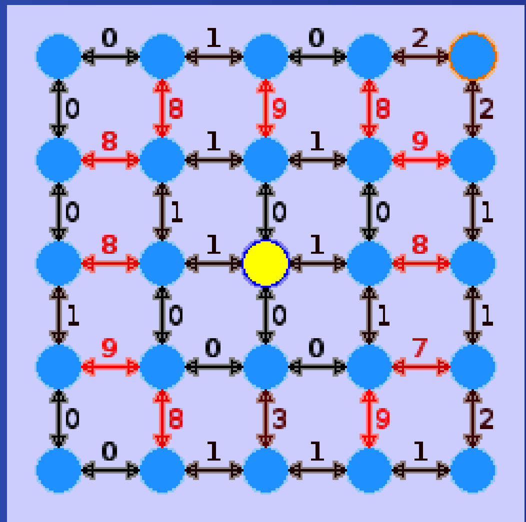
- They may conquer their adjacent nodes by offering them better paths.
- The process continues from the adjacent nodes in a non-decreasing (non-increasing) order of path value.

$$\text{if } f(\pi_s \cdot \langle s,t\rangle) < f(\pi_t) \ \text{ then } \pi_t \leftarrow \pi_s \cdot \langle s,t\rangle.$$

A simple example

An image-graph where arc weights $w(s,t)$ indicate the dissimilarity between adjacent nodes and we wish to compute V such that the roots of P belong to a seed set $\mathcal{S} = \{(3,3),(5,1)\}$.

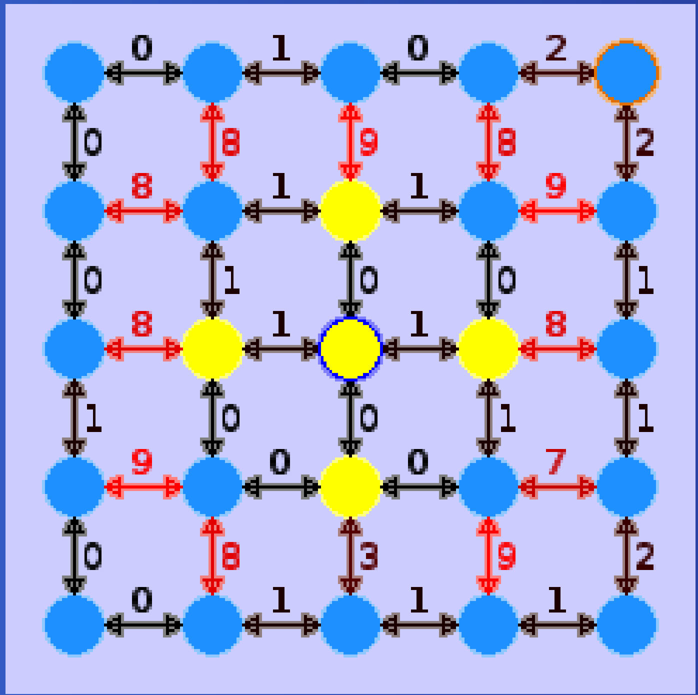


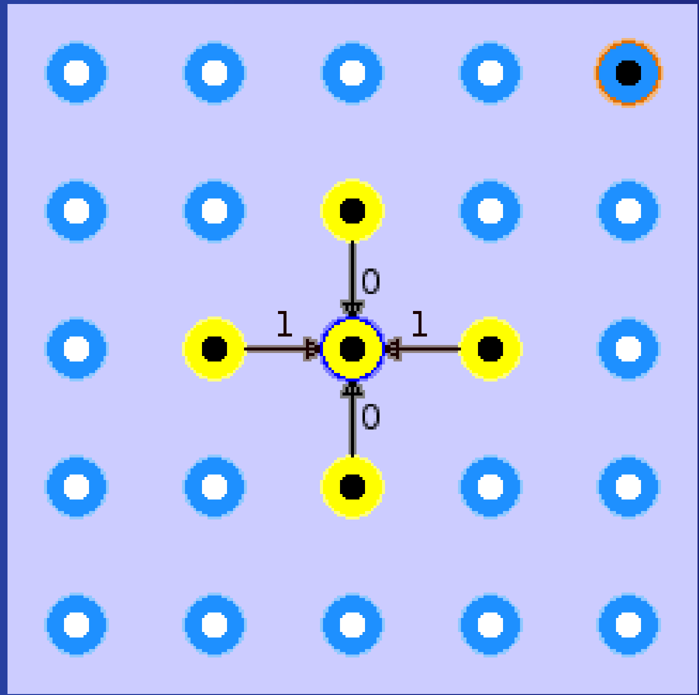
its path function

We need to define a path-initialization rule and a path-extension rule. Path function f_1 is then minimized.

$$
\begin{aligned}
f_1(\langle t\rangle) &= \begin{cases} 0 & \text{if } t \in \mathcal{S} \\ +\infty & \text{otherwise} \end{cases} \\
f_1(\pi_s \cdot \langle s, t\rangle) &= \max\{f_1(\pi_s), w(s,t)\}
\end{aligned}
$$

Path propagation





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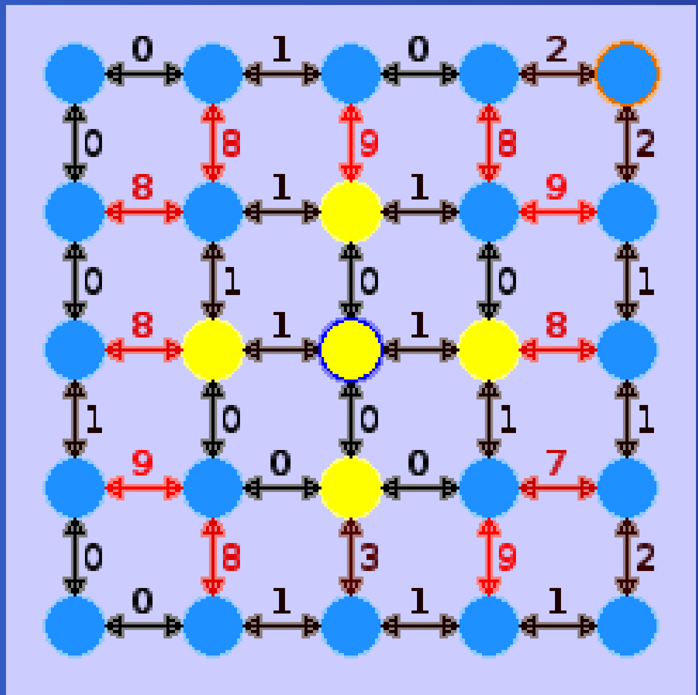

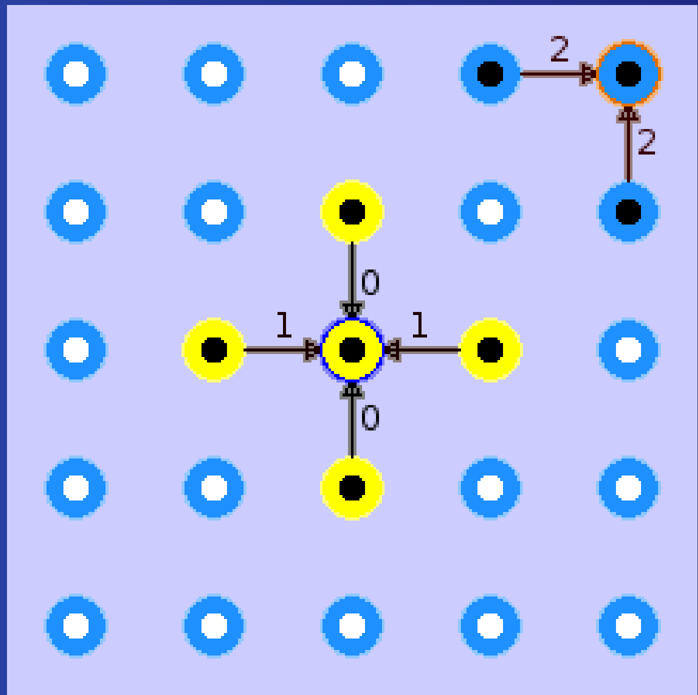

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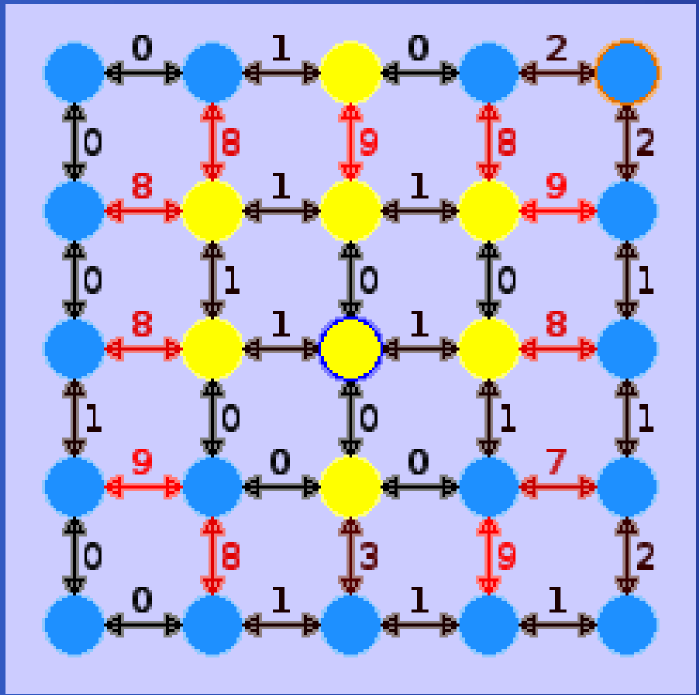

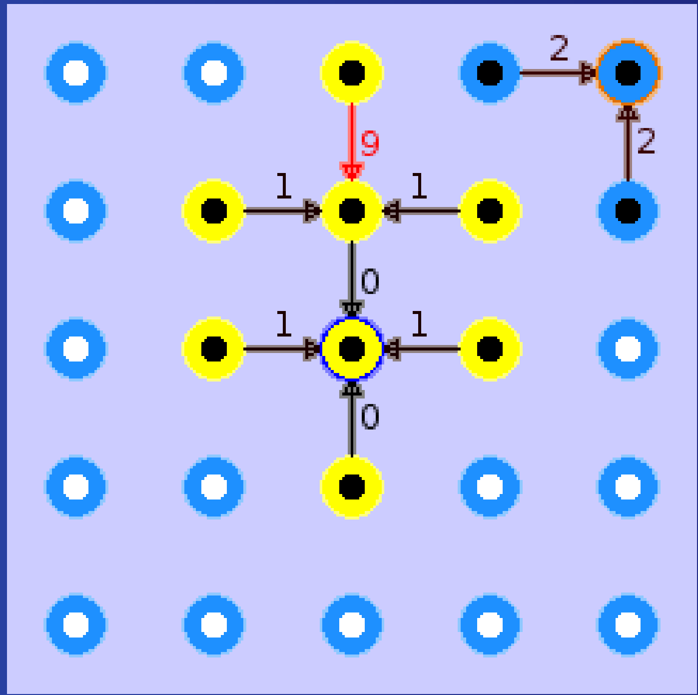

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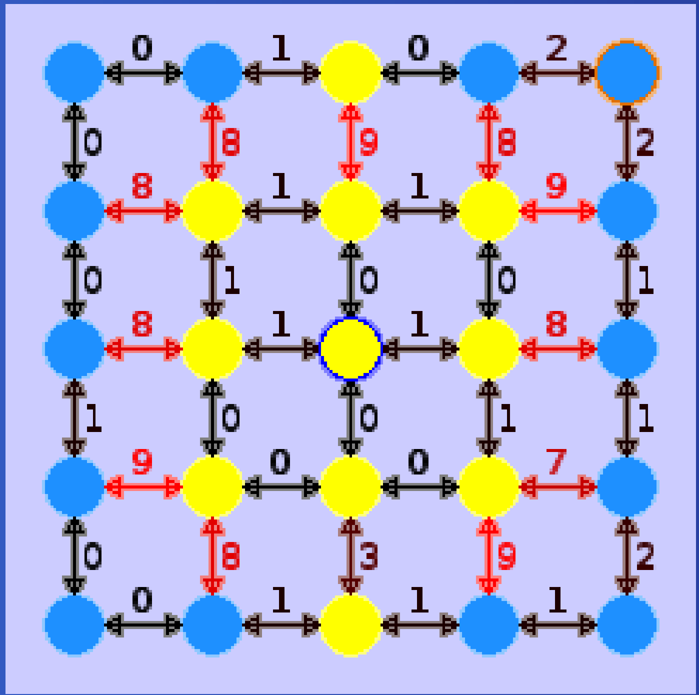

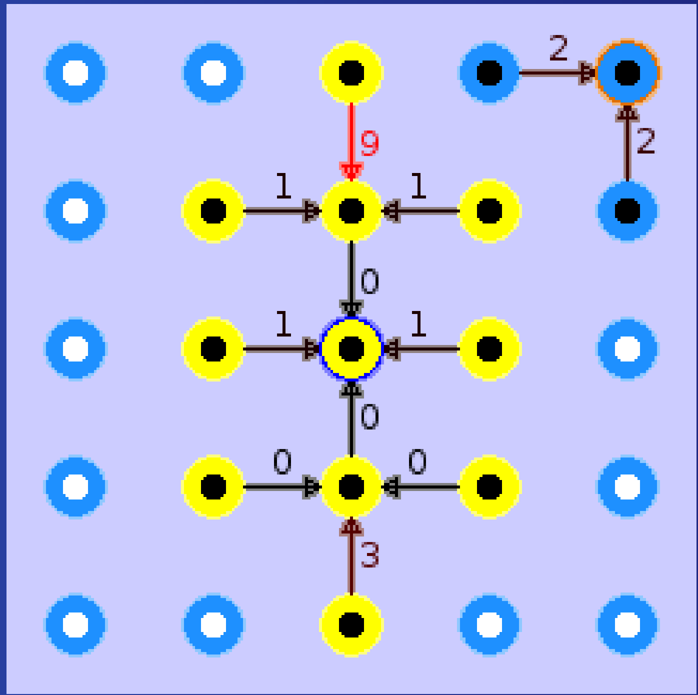

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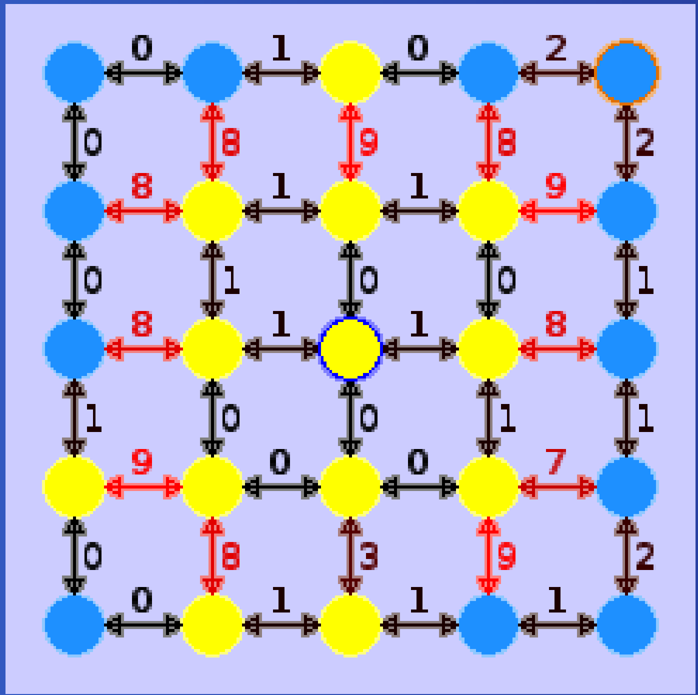

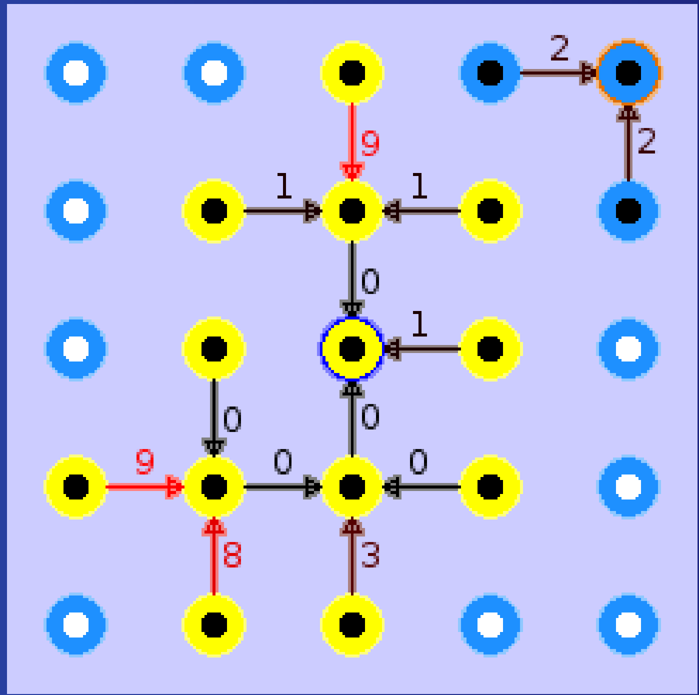

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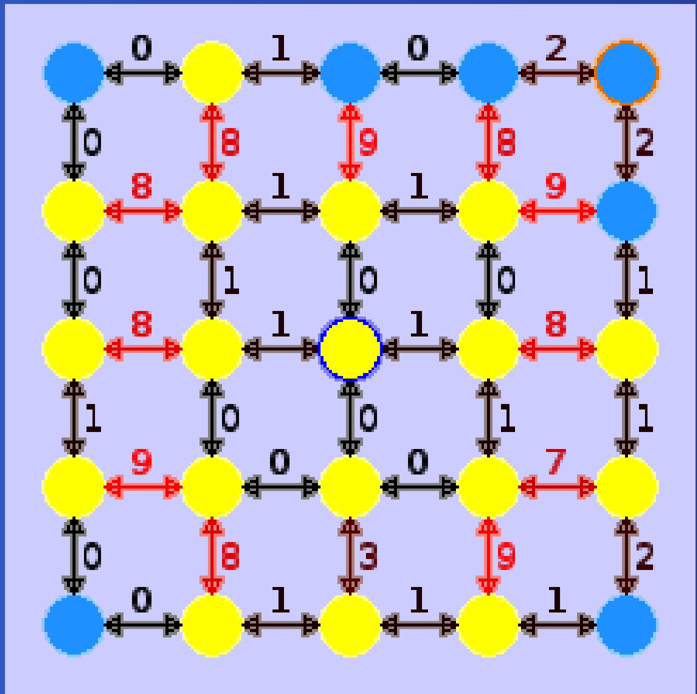
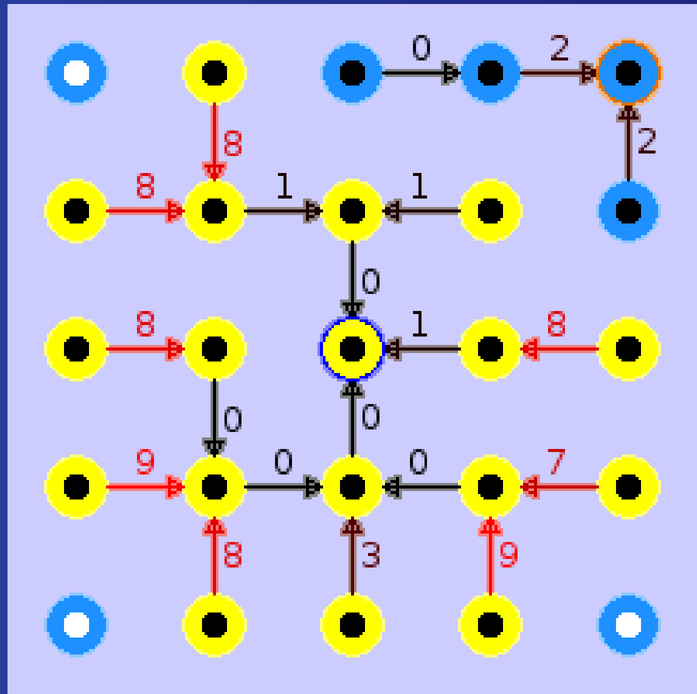
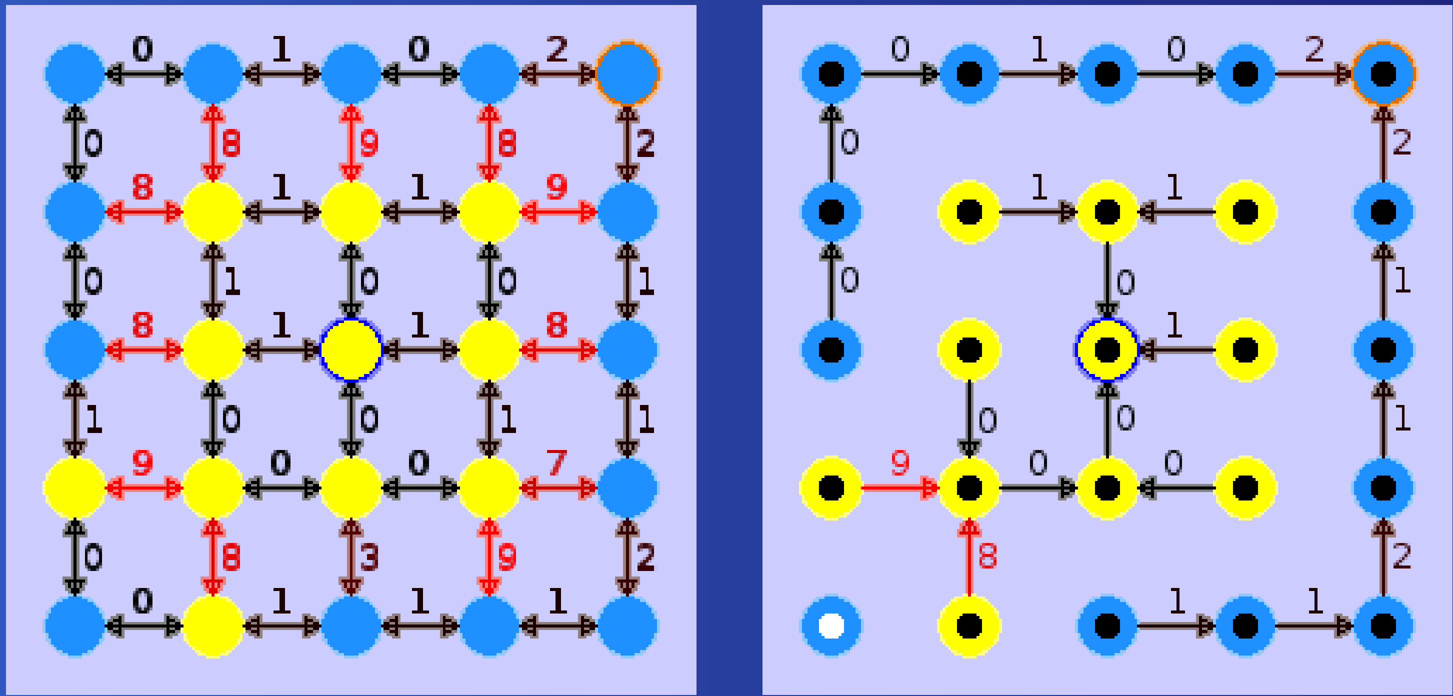




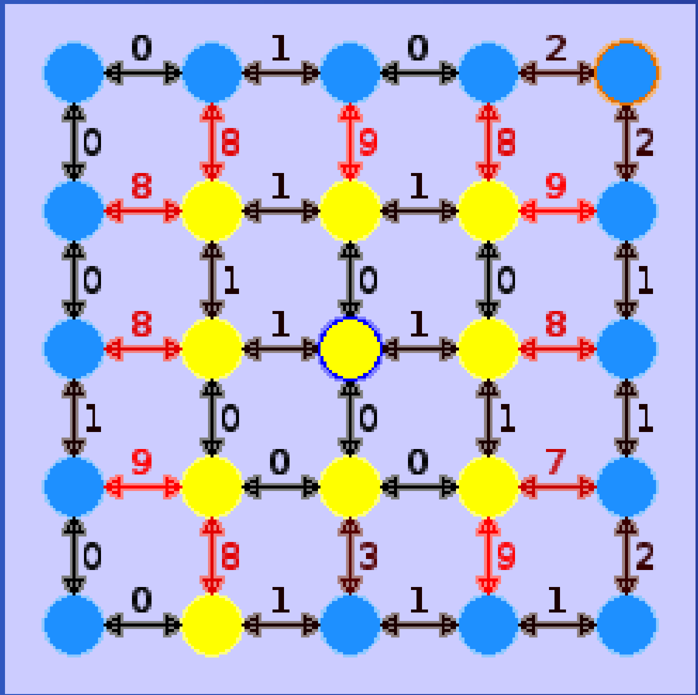
after 12 iterations.

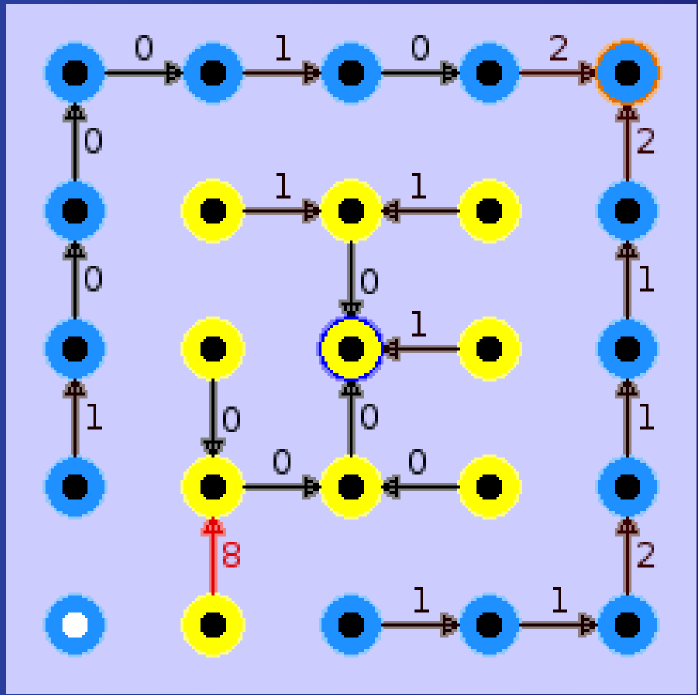
Path propagation



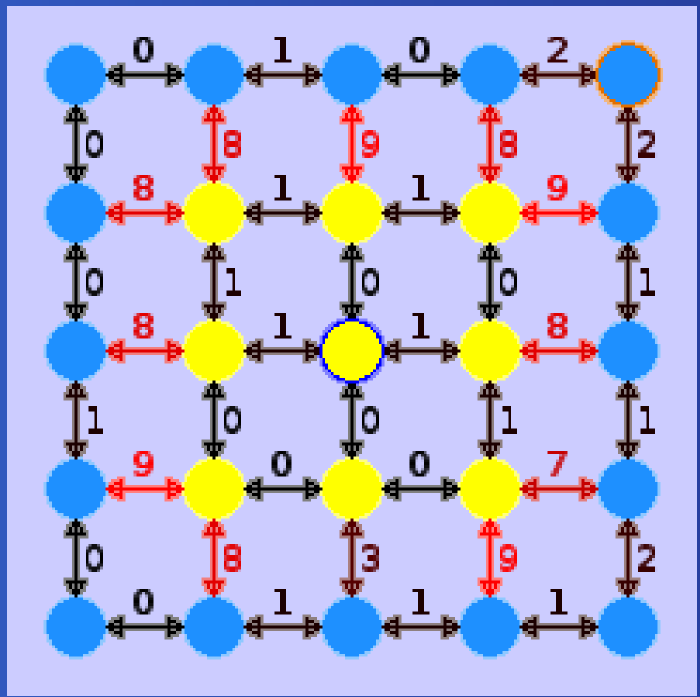
after 20 iterations.

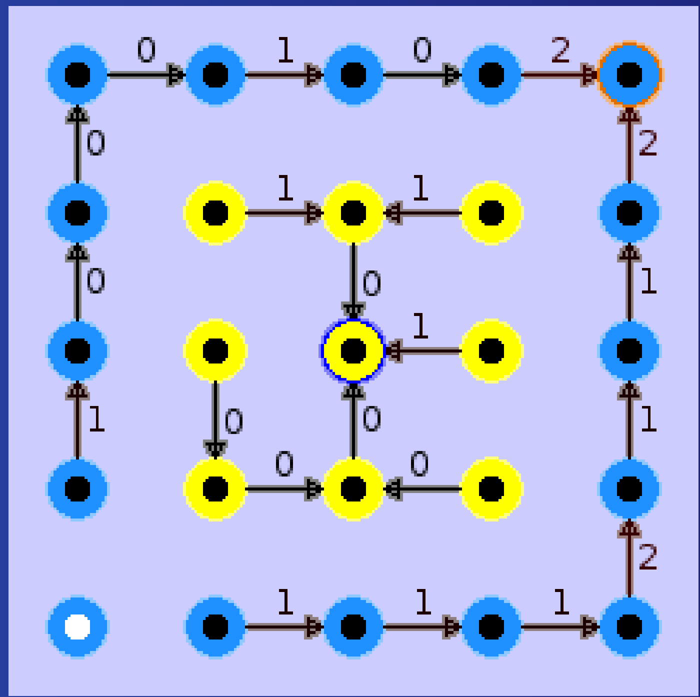
Path propagation





Path propagation





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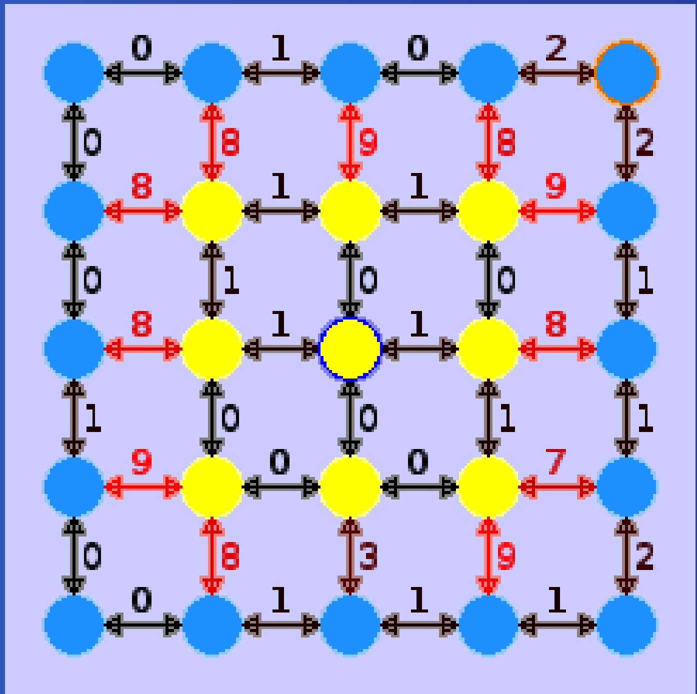

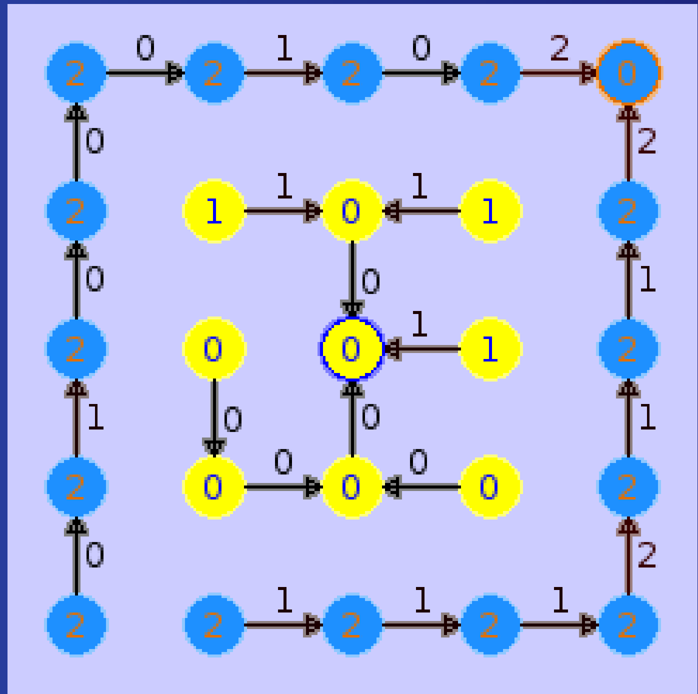

after 25 iterations.

Information propagation

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- The operators result from a local processing of one or more of these informations.

Its correctness

For every node t, there must exist at least one optimum path π_t which is either trivial or has the form $\pi_s \cdot \langle s, t \rangle$ where:

1. $f(\pi_s) \leq f(\pi_t)$.
2. π_s is optimum.
3. For any optimum path τ_s, $f(\tau_s \cdot \langle s, t \rangle) = f(\pi_t)$.

These conditions are applied to only optimum paths.

Euclidean distance transform (EDT)

The EDT of a pixel set $\mathcal{S}$ uses Euclidean adjacency $\mathcal{A}$ and minimizes path function f_2 [19].

$$f_2(\langle t \rangle) = \begin{cases} 0 & \text{if } t \in \mathcal{S} \\ +\infty & \text{otherwise} \end{cases}$$

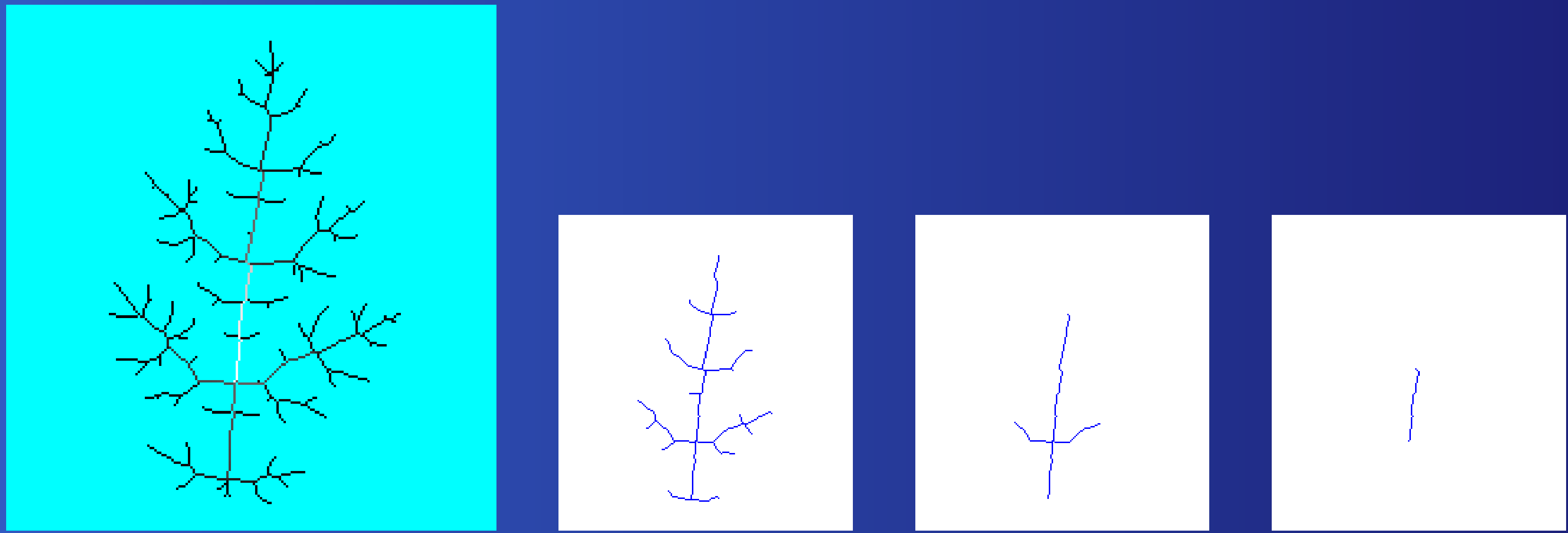
$$f_2(\pi_s \cdot \langle s, t \rangle) = \|t - R(s)\|$$

where $R(s)$ is the root of s.

EDT with Label Propagation

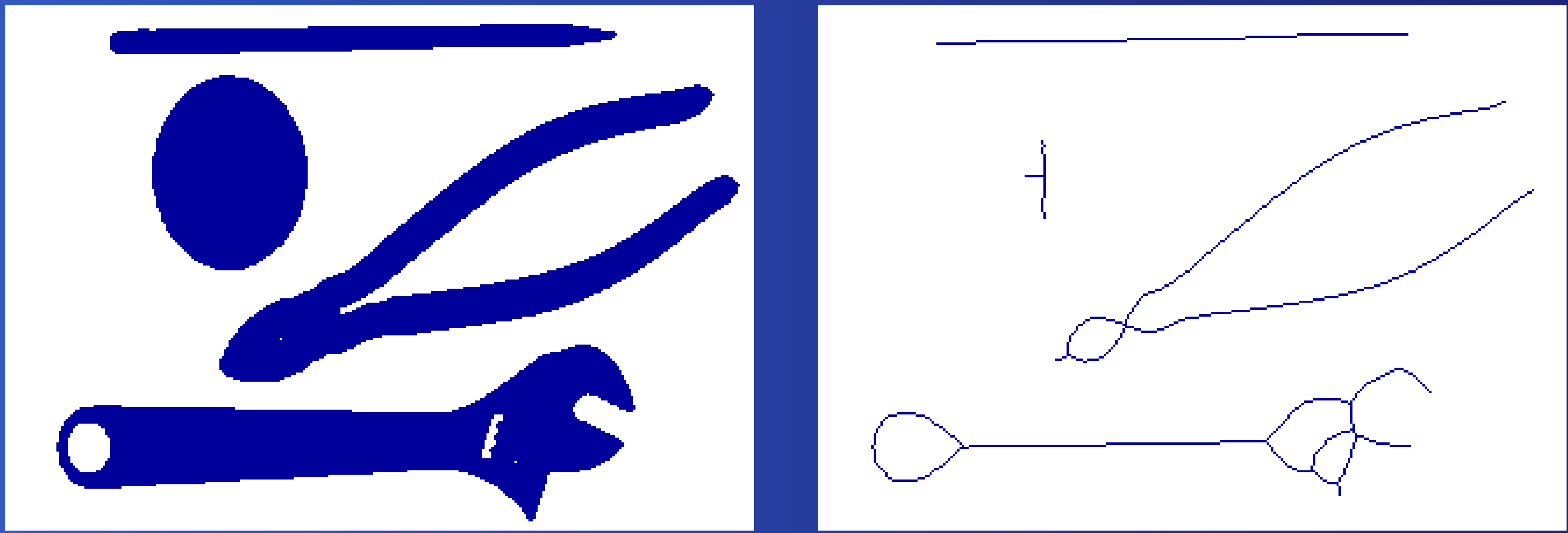
A consecutive integer number can be assigned to each contour pixel in $\mathcal{S}$ and propagated to the rest of the image.

Multiscale skeletons



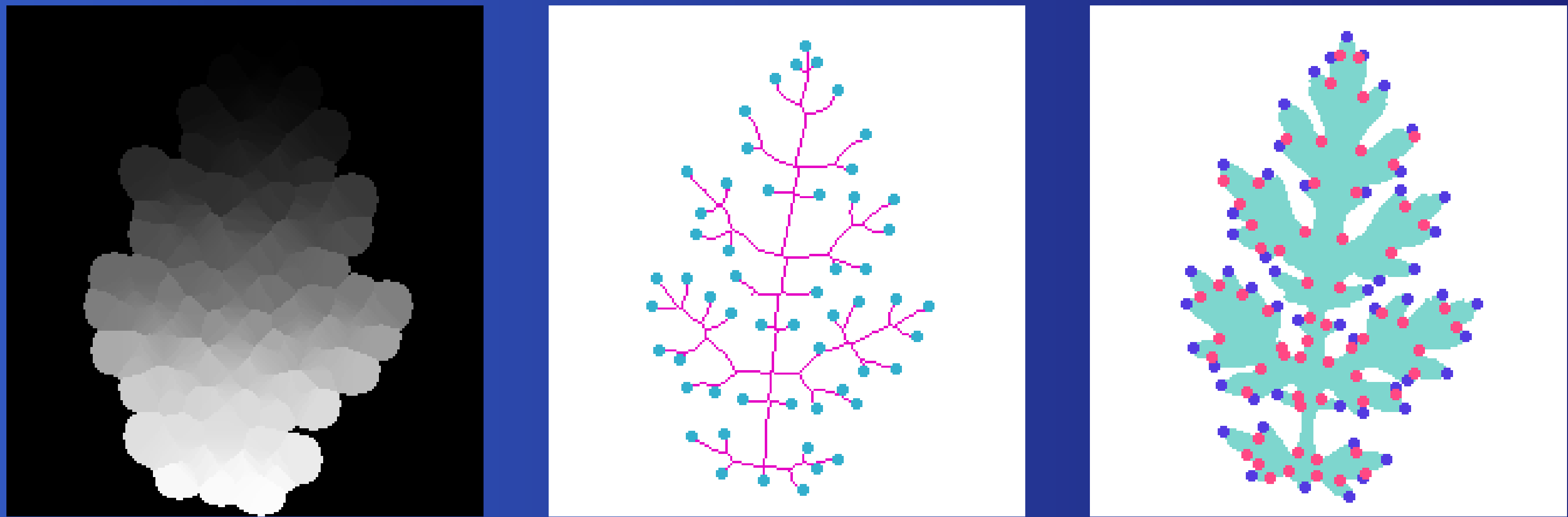
A difference image is obtained from the labeled image. Increasing thresholds create more simplified one-pixel-wide and connected skeletons.

Skeletons of multiples contours



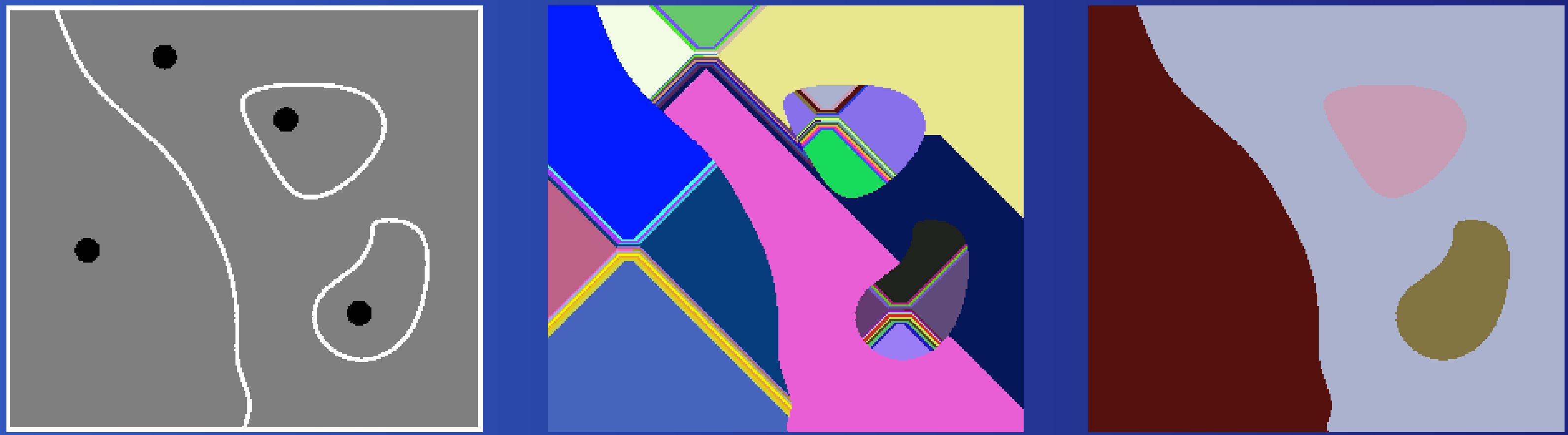
The method is easily extended to incorporate the SKIZ in the case of multiple contours.

Shape saliences



Skeleton saliences are detected from the aperture angles of their influence zones within a small dilation radius, leading to contour saliences [7].

Watershed from regional minima



We want to propagate a distinct label for each minimum. This can be done by forcing a single root in each minimum.

its path function

The minimization of path function f_3 forces a single root per minimum.

$$
\begin{aligned}
f_3(\langle t \rangle) &= \begin{cases} I(t) & \text{if } t \in \mathcal{R} \\ I(t)+1 & \text{otherwise} \end{cases} \\
f_3(\pi_s \cdot \langle s, t \rangle) &= \max\{f_3(\pi_s), I(t)\}
\end{aligned}
$$

where $I(t)$ is the image value of t and $\mathcal{R}$ is the root set identified on-the-fly by: if $P(s) = nil$ when s is removed from Q then $s \in \mathcal{R}$. Note that V(t)=I(t).

Superior reconstruction

The minimization of path function f_4 computes in V the superior reconstruction of a mask image $\hat{I} = (\mathcal{I}, I)$ from a marker image $\hat{H} = (\mathcal{I}, H)$ [3].

$$
\begin{aligned}
f_4(\langle t \rangle) &= \begin{cases} I(t) & \text{if } t \in \mathcal{R} \\ H(t) & \text{otherwise} \end{cases} \\
f_4(\pi_s \cdot \langle s, t \rangle) &= \max\{f_4(\pi_s), I(t)\}
\end{aligned}
$$

where $I(t) < H(t)$ for all $t \in \mathcal{I}$. We are forcing one root in R for each minimum of V.

Watershed from gray-scale marker

Simultaneously, the IFT with f_4 computes the watershed from the minima of V in a label map [3, 4].

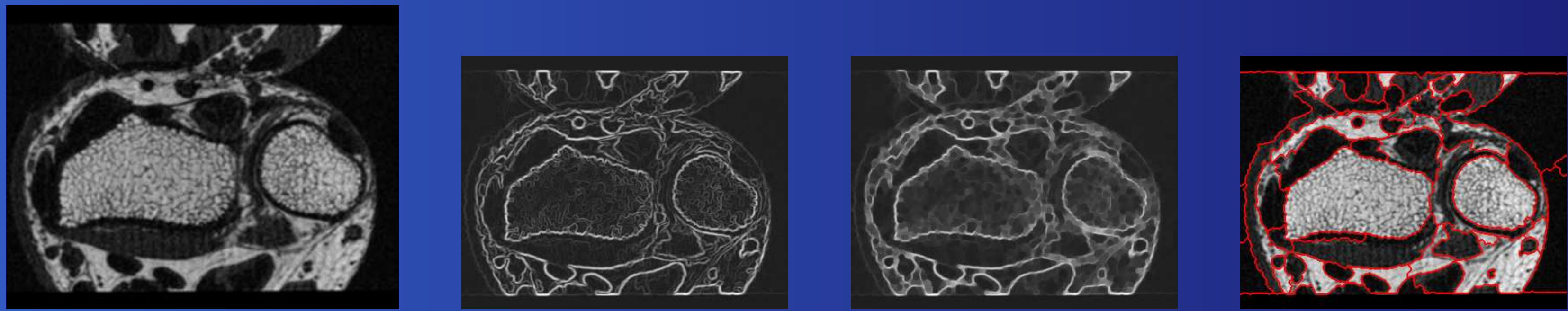


Image $\hat{H}$ is the closing with $d_i = 2.5$ on gradient $\hat{I}$, plus 1 ($d_i = 3.5$ for the IFT).

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- A pair $(\vec{v}, d)$ defines a descriptor for the data distribution in the feature (metric) space.

Adjacency relation for data clustering

The adjacency relation $\mathcal{A}_k$ interprets the dataset as a k-nn graph.

$$\mathcal{A}_k \quad : \quad (s,t) \in \mathcal{A}_k \text{ (or } t \in \mathcal{A}_k(s)\text{) if } t \text{ is } k \text{ nearest neighbor of } s \text{ in the feature space.}$$

The best value of k is obtained by minimizing a graph-cut measure on the data clustering results for increasing values of k [11].

The k-nn graph

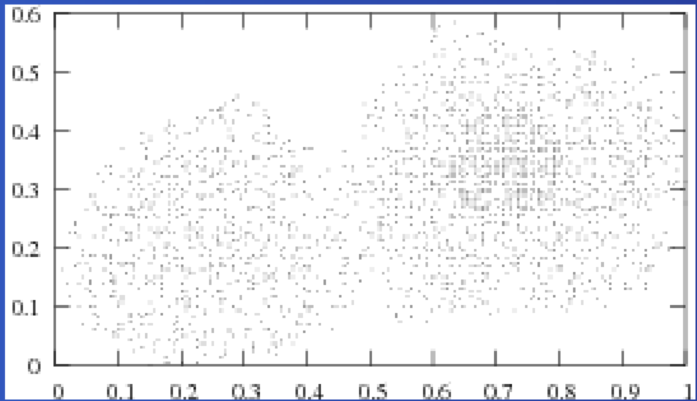
The graph is weighted on the arcs $(s,t) \in \mathcal{A}_k$ by $d(s,t)$ and on the nodes by a probability density function (pdf) ρ.

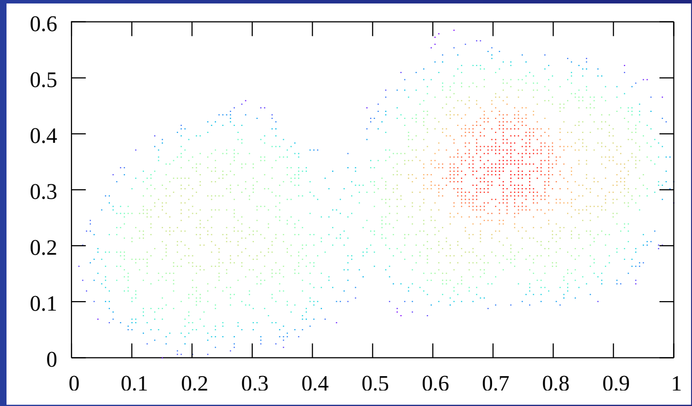
$$\rho(s) \quad = \quad \frac{1}{\sqrt{2\pi\sigma^2}|\mathcal{A}_k(s)|} \sum_{\forall t \in \mathcal{A}_k(s)} \exp\left(\frac{-d^2(s,t)}{2\sigma^2}\right)$$

where $\sigma = \frac{d_f}{3}$ and $d_f = \max_{\forall (s,t) \in \mathcal{A}_k}\{d(s,t)\}$.

An example of PDF

Data samples of connected clusters in a 2D feature space and their pdf.





The pdf varies from blue to red. The influence zones of its maxima define clusters (the mean-shift approach).

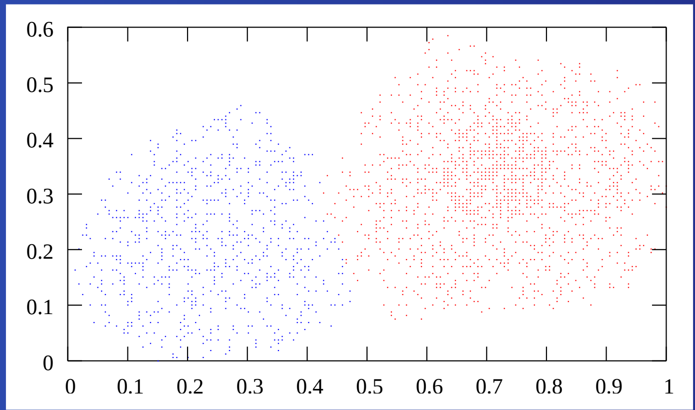
The mean-shift approach

The mean-shift algorithm computes the influence zones by following, for each sample s, the direction of the gradient of the pdf towards the steepest maximum around s. It is more robust to maximize f_5 with the IFT algorithm.

$$
\begin{aligned}
f_5(\langle t \rangle) &= \begin{cases} \rho(t) & \text{if } t \in \mathcal{R} \\ \rho(t) - \delta & \text{otherwise} \end{cases} \\
f_5(\pi_s \cdot \langle s, t \rangle) &= \min\{f_5(\pi_s), \rho(t)\} \\
\delta &= \min_{\forall (s,t) \in \mathcal{A}_k | \rho(t) \neq \rho(s)} |\rho(t) - \rho(s)|.
\end{aligned}
$$

the result is

the dual of the IFT with f_3 on ρ (watershed from regional minima).



The dual of the IFT with f_4 on ρ (watershed from gray-scale marker) can do better, by reducing the number of irrelevant clusters in real applications.

Data clustering by IFT

We then use path function f_6 for data clustering.

$$\begin{aligned} f_6(\langle t \rangle) &= \begin{cases} \rho(t) & \text{if } t \in \mathcal{R} \\ H(t) & \text{otherwise} \end{cases} \\ f_6(\pi_s \cdot \langle s, t \rangle) &= \min\{f_6(\pi_s), \rho(t)\} \end{aligned}$$

where $\rho(t) > H(t)$ and $H(t)$ can be the result of some anti-extensive operation on ρ, minus δ. We may scale ρ in $[1, K]$ and set $\delta = 1$.

Application to image segmentation

- The feature vectors $\vec{v}(s)$ can be created by a sequence of ASF by reconstruction for increasing values of d_i, and $d(s,t)$ may be $\|\vec{v}(t) - \vec{v}(s)\|$.

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- The best k-nn graph is usually impractical when the image pixels are the nodes of the graph.
- However, we can reduce the image scale and use its best k-nn graph to find a good d_f for pdf computation.

The pdf computation for images

The adjacency relation $\mathcal{A}$ and $\rho(s)$ are defined by

$$\begin{aligned}
\mathcal{A} &: (s,t) \in \mathcal{A} \text{ if } d(s,t) \leq d_f \text{ and } \|t-s\| \leq d_i, \\
\rho(s) &= \frac{1}{\sqrt{2\pi\sigma^2}|\mathcal{A}(s)|} \sum_{\forall t \in \mathcal{A}(s)} \exp\left(\frac{-d^2(s,t)}{2\sigma^2}\right)
\end{aligned}$$

where $\sigma = \frac{d_f}{3}$ and $d_i = 5.0$ is usually fine.

A few results

- Most image objects can be either correctly segmented or divided into a few regions.

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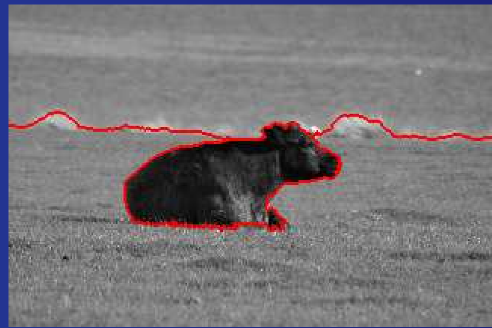
- Most image objects can be either correctly segmented or divided into a few regions.
- An anti-extensive operation on ρ (e.g., volume/area opening in the graph) is required in most cases to eliminate the influence zones of irrelevant maxima.

Running dog

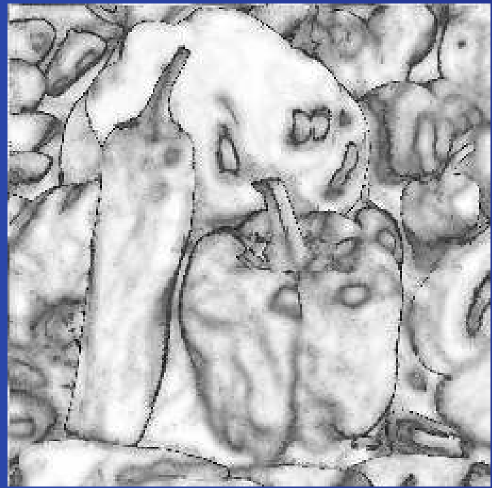
  

Resting cow/bull

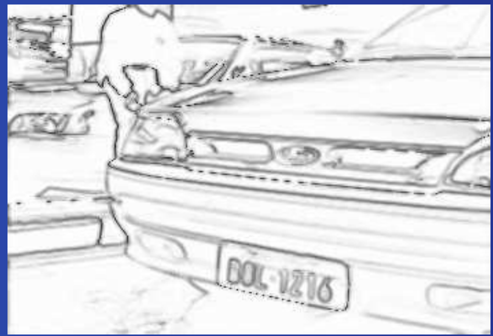


Dreaming house

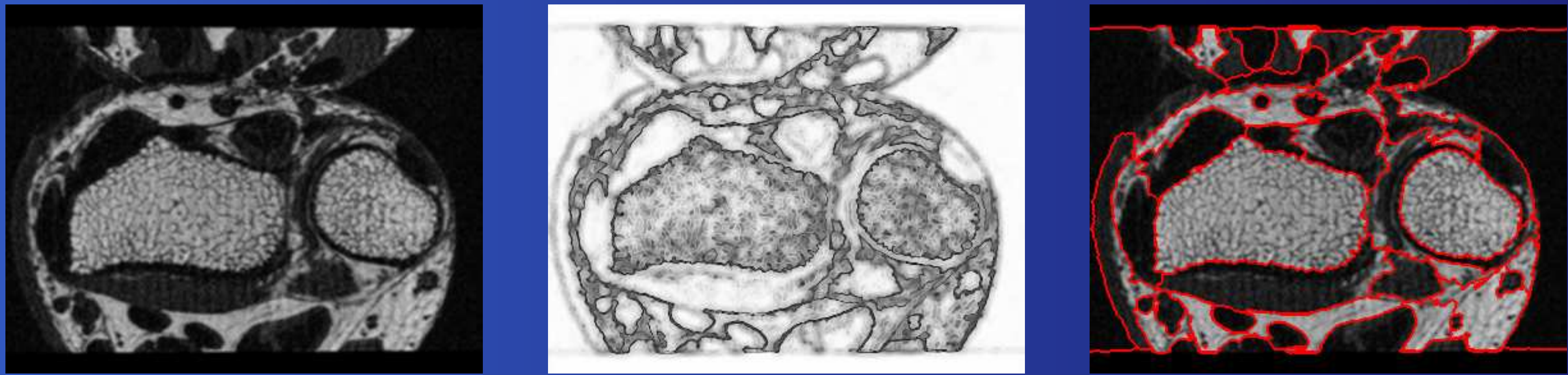
Colored peppers



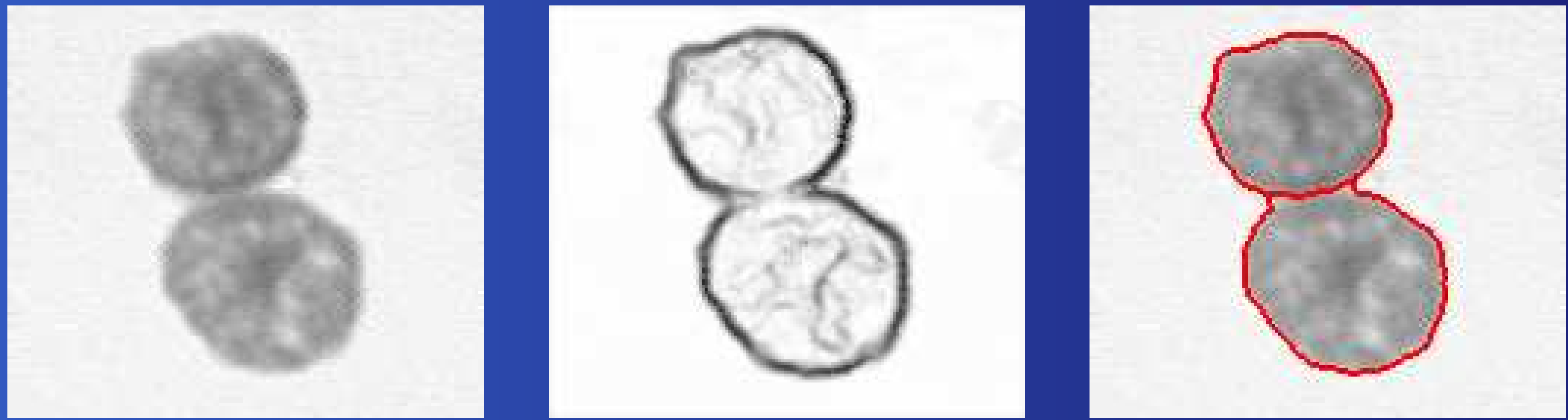
Car plate




MR wrist

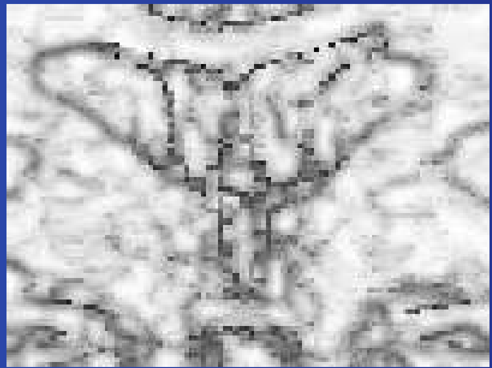


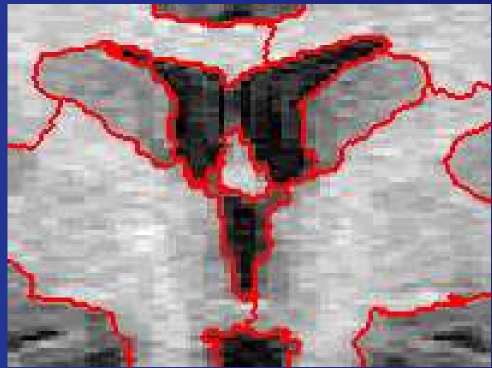
Connected cells



CT knee

MR brain





Some on-going works

- The IFT architecture [2].

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- The partition IFT [14].

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Some on-going works

- The IFT architecture [2].
- The partition IFT [14].
- Shape models to guide image segmentation by IFT.
- Supervised pattern classification based on IFT and pdf.

Some open problems

- How many ways one can combine object models with IFT operators for more effective image segmentation?

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- Can we do image compression using the IFT?

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- Can we do image compression using the IFT?
- How to devise new image operators using sequences of differential IFTs [13]?

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- How many ways one can combine object models with IFT operators for more effective image segmentation?
- Can we do image compression using the IFT?
- How to devise new image operators using sequences of differential IFTs [13]?
- Can we develop super resolution techniques using the IFT-classifiers?

Acknowledgments

Thank you!!!

FAPESP, CNPq, and the program chairs and organization committee of the ISMM'07.

References

[1] A.X. Falcão, J. Stolfi, and R.A. Lotufo, "The image foresting transform: Theory, algorithms, and applications," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 26, no. 1, pp. 19–29, 2004.

[2] F. Cappabianco, G. Araújo, and A.X. Falcão, "The image foresting transform architecture," in *IEEE Intl. Conf. on Field Programmable Technology (ICFPT)*, Kokurakita, Kitakyushu, Japan, Dec 2007, IEEE, to appear.

[3] A.X. Falcão, B. S. da Cunha, and R. A. Lotufo, "Design of connected operators using the image foresting transform," in *SPIE on Medical Imaging*, Feb 2001, vol. 4322, pp. 468–479.

[4] R.A. Lotufo, A.X. Falcão, and F. Zampirolli, "IFT-Watershed from gray-scale marker," in *XV Brazilian Symp. on Computer Graphics and Image Processing (SIBGRAPI)*. Oct 2002, pp. 146–152, IEEE.

[5] R. Audigier and R.A. Lotufo, "Seed-relative segmentation robustness of watershed and fuzzy connectedness approaches," in *XX Brazilian Symposium on Computer Graphics and Image Processing (SIBGRAPI)*, Belo Horizonte, MG, Oct 2007, pp. 61–68, IEEE CPS.

[6] R. Audigier and R.A. Lotufo, "Watershed by image foresting transform, tie-zone, and theoretical relationship with other watershed definitions," in *Mathematical Morphology and its Applications to Signal and Image Processing (ISMM)*, Rio de Janeiro, RJ, Oct 2007, pp. 277–288, MCT/INPE.

[7] R.S. Torres, A.X. Falcão, and L.F. Costa, "A graph-based approach for multiscale shape analysis," *Pattern Recognition*, vol. 37, no. 6, pp. 1163–1174, 2004.

[8] F.P.G. Bergo, A.X. Falcão, P.A.V. Miranda, and L.M. Rocha, "Automatic image segmentation by tree pruning," *Journal of Mathematical Imaging and Vision*, 2007, to appear.

[9] A.X. Falcão, P.A.V. Miranda, and A. Rocha, "A linear-time approach for image segmentation using graph-cut measures," in *8th Intl. Conf. on Advanced Concepts for Intelligent Vision Systems (ACIVS)*, Antwerp, Belgium, 2006, vol. LNCS 4179, pp. 138–149, Springer.

[10] A.X. Falcão, P.A.V. Miranda, A. Rocha, and F.P.G. Bergo, "Object detection by κ-connected seed competition," in *XVIII Brazilian Symp. on Computer Graphics and Image Processing (SIBGRAPI)*, Natal, RN, Oct 2005, pp. 97–104, IEEE CPS.

[11] L.M. Rocha, A.X. Falcão, and L. Meloni, "Data clustering based on optimum-path forest and probability density function," Tech. Rep. IC-07-031, State University of Campinas, Institute of Computing, 2007.

[12] J.P. Papa, A.X. Falcão, P.A.V. Miranda, C.T.N. Suzuki, and N.D.A. Mascarenhas, "Design of robust pattern classifiers based on optimum-path forests," in *Mathematical Morphology and its Applications to Signal and Image Processing (ISMM)*, Rio de Janeiro, RJ, Oct 2007, pp. 337–348, MCT/INPE.

[13] A. X. Falcão and F. P. G. Bergo, "Interactive volume segmentation with differential image foresting transforms," *IEEE Trans. on Medical Imaging*, vol. 23, no. 9, pp. 1100–1108, 2004.

[14] F.P.G. Bergo and A.X. Falcão, "A partitioned algorithm for the image foresting transform," in *Mathematical Morphology and its Applications to Signal and Image Processing (ISMM)*, Rio de Janeiro, RJ, Oct 2007, pp. 425–436, MCT/INPE.

[15] I. Ragnemalm, "Fast erosion and dilation by contour processing and thresholding of distance maps," *Pattern Recognition Letters*, vol. 13, pp. 161–166, Mar 1992.

[16] A.X. Falcão, J.K. Udupa, and F.K. Miyazawa, "An ultra-fast user-steered image segmentation paradigm: Live-wire-on-the-fly," *IEEE Trans. on Medical Imaging*, vol. 19, no. 1, pp. 55–62, Jan 2000.

[17] L.G. Nyúl, A.X. Falcão, and J.K. Udupa, "Fuzzy-connected 3D image segmentation at interactive speeds," *Graphical Models*, vol. 64, no. 5, pp. 259–281, 2003.

[18] R. Audigier, R.A. Lotufo, and A.X. Falcão, "3D visualization to assist iterative object definition from medical images," *Computerized Medical Imaging and Graphics*, vol. 30, no. 4, pp. 217–230, Jun 2006.

[19] A.X. Falcão, L.F. Costa, and B.S. da Cunha, "Multiscale skeletons by image foresting transform and its applications to neuromorphometry," *Pattern Recognition*, vol. 35, no. 7, pp. 1571–1582, Apr 2002.

[20] R.S. Torres and A.X. Falcão, "Contour salience descriptors for effective image retrieval and analysis," *Image and Vision Computing*, vol. 25, no. 1, pp. 3–13, Jan 2007.

[21] F.A. Andaló, P.A.V. Miranda, R.S. Torres, and A.X. Falcão, "Detecting contour saliences using tensor scale," in *14th IEEE Intl.*

Conf. on Image Processing, San Antonio, Texas, Sep 2007, vol. VI, pp. 349–352, IEEE SPS.

[22] F.A. Andaló, P.A.V. Miranda, R.S. Torres, and A.X. Falcão, "A new shape descriptor based on tensor scale," in *Mathematical Morphology and its Applications to Signal and Image Processing (ISMM)*, Rio de Janeiro, RJ, Oct 2007, pp. 141–152, MCT/INPE.

[23] R.A. Lotufo and A.X. Falcão, "The ordered queue and the optimality of the watershed approaches," in *Mathematical Morphology and its Applications to Image and Signal Processing (ISMM)*, vol. 18, pp. 341–350. Kluwer, Jun 2000.

[24] A.X. Falcão and J.K. Udupa, "A 3D generalization of user-steered live wire segmentation," *Medical Imaging Analysis*, vol. 4, no. 4, pp. 389–402, Dec 2000.

[25] P. A. V. Miranda, F. P. G. Bergo, L. M. Rocha, and A. X. Falcão, "Tree-pruning: A new algorithm and its comparative analysis with the watershed transform for automatic image segmentation," in *XIX Brazilian Symp. on Computer Graphics and Image Processing (SIBGRAPI)*. Oct 2006, pp. 37–44, IEEE CPS.

[26] A. X. Falcão, F. P. G. Bergo, and P. A. V. Miranda, "Image segmentation by tree pruning," in *XVII Brazilian Symp. on Computer Graphics and Image Processing (SIBGRAPI)*. Oct 2004, pp. 65–71, IEEE CPS.

[27] J.A. Montoya-Zegarra, J.P. Papa, N.J. Leite, R.S. Torres, and A.X. Falcão, "Rotation-invariant texture recognition," in *3rd Intl. Symp. on Visual Computing*, Lake Tahoe, Nevada, CA, USA, Nov 2007, vol. Part II, LNCS 4842, pp. 193–204, Springer.