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Wilson F. N. Santos

*Combustion and Propulsion Laboratory
National Institute for Space Research
Cachoeira Paulista – SP, 12630–000 BRAZIL*

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Cachoeira Paulista, SP 12630-000 BRAZIL*

Computations using the Direct Simulation Monte Carlo method are presented for hypersonic flow over flat-nose power-law leading edges. The primary aim of this paper is to examine the geometry effect of such leading edges on the shock wave structure. The impact of the nose-thickness and afterbody-shape variations on shock wave shape, shock standoff distance, and shock thickness of such leading edges is investigated by using a model that classifies the molecules in three distinct classes: “undisturbed freestream”, “reflected from the boundary” and “scattered”, i.e., molecules that had been indirectly affected by the presence of the leading edge. Comparisons based on shock wave standoff distance and shock wave thickness are made between these new blunt configurations and circular cylinder shape, usually assumed as the appropriate blunting geometry for heat transfer considerations. It was found that the new blunt leading edges provided smaller shock wave standoff distance and shock wave thickness, compared to the corresponding circular cylinder.

Nomenclature

a	Constant in the body equation, Eq.(1)
A, B	Constant in the shock equation, Eq.(2)
C, D	Constant in the body equation, Eq.(3)
H	Body height at the base, m
Kn	Knudsen number, λ/l
L	Body length, m
l	Characteristic length, m
M	Mach number
m	Molecular mass, m
n	Afterbody power law exponent, number density, m^{-3}
p	Body power law exponent
q	Shock power law exponent
R	Circular cylinder radius, m
Re	Reynolds number, $\rho V l / \mu$
s	Arc length, m
T	Temperature, K
t	Leading edge thickness, m
V	Velocity, m/s
x, y	Cartesian axes in physical space, m

*Researcher, Combustion and Propulsion Laboratory. AIAA Member.

y_{nose}	Half thickness of the body nose, m
δ	Shock wave thickness, m
Δ	Shock wave standoff distance, m
η	Coordinate normal to body surface, m
θ	Wedge half angle, deg
λ	Mean free path, m
ξ	Coordinate tangent to body surface, m
ρ	Density, kg/m ³
<i>Subscript</i>	
I, II, III	Refer to the molecule class
w	Wall conditions
∞	Freestream conditions

I. Introduction

THE successful design of high-lift, low-drag hypersonic configurations will depend on the ability to incorporate relatively sharp leading edges that combine good aerodynamic properties with acceptable heating rates. Certain configurations, such as hypersonic waveriders¹, are designed analytically with infinitely sharp leading edge for shock wave attachment. The shock wave acts as a barrier in order to prevent spillage of higher pressure airflow from the lower side of the vehicle to the upper side, resulting in a high-pressure differential and enhanced lift. Nevertheless, for practical applications, these sharp leading edges must be blunted for heat transfer, manufacturing, and handling concerns, with associated departures from ideal performance. Usually, a round leading edge with constant radius of curvature near the stagnation point has been chosen for this purpose. However, cylindrical bluntness added for heating rate reduction will tend to displace the shock wave. In addition, the shock detachment distance on a cylinder, with associated leakage, scales with the radius of curvature.

Certain classes of non-circular shapes may provide the required bluntness with smaller shock separation than round leading edges, thus allowing manufacturing, and ultimately heating control, with reduced departures from ideal aerodynamic performance. For this purpose, theoretical and analytical studies^{2–12} have recently focused the attention to power-law shapes ($y \propto x^p, 0 < p < 1$). The major interest in these works has gone into considering the power-law shape as possible candidates for blunting geometries of hypersonic leading edges, such as hypersonic waverider vehicles¹.

Of particular interest on power-law shapes is the work by Santos and Lewis⁴. For the idealized situation of two-dimensional rarefied hypersonic flow, they have investigated the sensitivity of the stagnation point heating and total drag to shape variations of such leading edges. The emphasis of the work was to compare power law leading edges with round leading edges (circular cylinder) in order to determine which geometry would be better suited as a blunting profile in terms of stagnation point heating and total drag coefficient. Their analysis showed that power law leading edges provided smaller total drag and larger stagnation point heating than the circular cylinder under the range of conditions investigated.

In an effort to improve the stagnation point heating, Santos¹³ introduced a modification into the power law shaped leading edge. The modified leading edge was composed by a flat nose supplemented by an afterbody surface defined by a power-law shape, named flat-nose power-law leading edge. This modification was based on the work of Reller¹⁴, who showed that a method of designing low heat transfer bodies is devised on the premise that the rate of heat transfer to the nose will be low if the local velocity is low, while the rate of heat transfer to the afterbody will be low if the local density is low. A typical body resulting from this design method consists of a flat nose followed by a highly curved, but for the most part slightly inclined, afterbody surface.

In this connection, Santos¹³ has examined the impact of the nose-thickness and afterbody-shape variations on the aerodynamic surface quantities over these flat-nose power-law leading edges. The thickness effect

was examined for a range of Knudsen number, based on the thickness of the flat nose, covering from the transitional flow regime to the free molecular flow regime. The emphasis of the work was to compare the heat transfer and total drag of this new contour with those obtained not only for zero-thickness power-law leading edges but also for round leading edges. It was found a significant reduction on the stagnation point heating and a slight increase in the total drag associated to this new contour for the conditions investigated.

In continuation of the flat-nose power-law shape investigation, the present work extends the analysis presented by Santos¹³ by examining computationally the shock wave structure over these contours. In the present account, the primary goal is to assess the sensitivity of the shock standoff distance, shock wave thickness and shock wave shape to variations in the thickness of the leading edge and on the afterbody shape, and to compare them to the round leading edges (circular cylinder). Comparisons based on shock standoff distance are made to evaluate the benefits and disadvantages of these new blunt shapes and to determine the appropriateness of using these shapes in hypersonic waverider design connected to high-altitude/low-density applications^{15–18}.

II. Leading Edge Geometry Definition

In dimensional form, the power law contours that define the shapes of the afterbody surfaces are given by the following expression,

$$y = y_{nose} + ax^n \quad (1)$$

where y_{nose} is the half thickness of the flat nose of the leading edges, n is the power law exponent and a is the power law constant which is a function of n .

The flat-nose power-law shapes are modeled by assuming a sharp leading edge (wedge) of half angle θ with a circular cylinder of radius R inscribed tangent to this wedge. The flat-nose power-law shapes, inscribed between the wedge and the cylinder, are also tangent to both shapes at the same common point where they have the same slope angle. It was assumed a leading edge half angle of 10 degree, a circular cylinder diameter of 10^{-2}m , power law exponents of $2/3$, 0.7 , $3/4$, and 0.8 , and frontal surface thicknesses t/λ_∞ of 0 , 0.01 , 0.1 and 1 , where $t = 2y_{nose}$ and λ_∞ is the freestream molecular mean free path. Figure 1 shows schematically this construction.

Figure 1. Drawing illustrating the leading edge geometries.

From geometric considerations, the power law constant a is obtained by matching slope on the wedge, circular cylinder and flat-nose power-law body at the tangency point. The common body height H at the tangency point is equal to $2R \cos \theta$, and the body length L , from the nose to the tangency point in the axis of symmetry, is given by $n(H - t)/2 \tan \theta$. Since the wake region behind the bodies is not of interest in this investigation, it was assumed that the leading edges are infinitely long but only the length L is considered.

III. Computational Method

The various phenomena found in high-speed flow are often described in terms of dimensionless parameters of the flow such as Reynolds number and Mach number. The former parameter may be considered to be a measure of the effect of viscosity and the latter parameter a measure of the effect of compressibility on

the flowfield. An additional parameter, the Knudsen number, becomes important when considering rarefied flow. The Knudsen number, $Kn = \lambda/l$, is defined as the ratio of the mean free path λ of the molecules to a characteristic body dimension l . Traditionally, flows are divided into four regimes¹⁹: $Kn < 0.01$, continuum flow, $0.01 < Kn < 0.1$, slip flow, $0.1 < Kn < 10$, transitional flow, and $Kn > 10$, free molecular flow.

Flow in the regime of intermediate Knudsen numbers, $0.01 < Kn < 10$, is difficult to deal with analytically and, at this time, it appears that the Direct Simulation Monte Carlo (DSMC) method pioneered by Bird²⁰ is conventionally regarded as being the most accurate and credible technique for computing leading edge flows in this flow regime.

In the DSMC code, the molecular collisions are modeled by using the variable hard sphere (VHS) molecular model²¹ and the no time counter (NTC) collision sampling technique²². The energy exchange between kinetic and internal modes is controlled by the Borgnakke-Larsen statistical model²³. Simulations are performed using a non-reacting gas model consisting of two chemical species, N_2 and O_2 . Energy exchanges between the translational and internal modes, rotational and vibrational, are considered. Relaxation collision numbers of 5 and 50 were used for the calculations of rotation and vibration, respectively.

In order to easily account for particle-particle collisions, the flowfield is divided into an arbitrary number of regions, which are subdivided into computational cells. The cells are further subdivided into subcells. The cell provides a convenient reference sampling of the macroscopic gas properties, while the collision partners are selected from the same subcell for the establishment of the collision rate.

The computational domain used for the calculation is made large enough so that body disturbances do not reach the upstream and side boundaries, where freestream conditions are specified. A schematic view of the computational domain is depicted in Fig. 2. Side I is defined by the body surface. Diffuse reflection with complete thermal accommodation is the condition applied to this side. Advantage of the flow symmetry is taken into account, and molecular simulation is applied to one-half of a full configuration. Thus, side II is a plane of symmetry. In such a boundary, all flow gradients normal to the plane are zero. At the molecular level, this plane is equivalent to a specular reflecting boundary. Side III is the freestream side through which simulated molecules enter and exit. Finally, the flow at the downstream outflow boundary, side IV, is predominantly supersonic and vacuum condition is specified²⁴. At this boundary, simulated molecules can only exit.

Figure 2. Schematic view of the computational domain.

Application of a numerical method to solve practical problems requires a reliable way in order to estimate the accuracy of the solution. The numerical accuracy in DSMC method depends on the grid resolution (cell size) chosen, on the time step as well as on the number of particles per computational cell.

In the DSMC algorithm, the linear dimensions of the cells should be small in comparison with the scale length of the macroscopic flow gradients normal to streamwise directions, which means that the cell dimensions should be of the order of or smaller than the local mean free path^{25–26}. The time step should be chosen to be sufficiently small in comparison with the local mean collision time^{27–28}. In general, the total simulation time, discretized into time steps, is identified with the physical time of the real flow. Finally, the number of simulated particles has to be large enough to make statistical correlations between particles insignificant.

These effects were investigated to determine the number of cells and the number of particles required to achieve grid independence solutions. Grid independence was tested by running the calculations with half and

double the number of cells in each direction compared to a standard grid. Solutions were near identical for all grids used and were considered fully grid independent. A discussion of both effects on the aerodynamic surface quantities for power law shapes with zero-thickness nose ($t/\lambda_\infty = 0$) is described in details in Santos and Lewis⁸. The same procedure was adopted for the $t/\lambda_\infty > 0$ cases. Nonetheless, the discussion will not be presented here.

The flow conditions used for the numerical simulation of flow past the leading edges are those given by Santos¹³ and summarized in Table 1, and the gas properties²⁰ are tabulated in Table 2. The freestream velocity V_∞ is assumed to be constant at 3.5 km/s, which corresponds to a freestream Mach number M_∞ of 12. The translational and vibrational temperatures in the freestream are in equilibrium at 220 K, and the leading edge surface has a constant wall temperature T_w of 880 K for all cases considered. The freestream Reynolds number by unit meter Re_∞ is 21455 based on conditions in the undisturbed stream.

For the thicknesses investigated, $t/\lambda_\infty = 0, 0.01, 0.1$ and 1, the overall Knudsen numbers correspond to $Kn_t = \infty, 100, 10$ and 1, respectively. It is important to mention that $Kn_t = \infty$ case corresponds to the power law leading edge set already investigated by Santos and Lewis⁴.

Table 1. Freestream flow conditions

Altitude (km)	T_∞ (K)	p_∞ (N/m ²)	ρ_∞ (kg/m ³)	μ_∞ (Ns/m ²)	n_∞ (m ⁻³)	λ_∞ (m)
70	220.0	5.582	8.753×10^{-5}	1.455×10^{-5}	1.8209×10^{21}	9.03×10^{-4}

Table 2. Gas properties

	X	m	d	ω
	mole fraction	molecular mass, kg	molecular diameter, m	viscosity index
O_2	0.237	5.312×10^{-26}	4.01×10^{-10}	0.77
N_2	0.763	4.650×10^{-26}	4.11×10^{-10}	0.74

IV. Computational Procedure

The problem of predicting the shape and location of detached shock waves has been stimulated by the necessity for blunt noses and leading edges configurations designed for hypersonic flight in order to cope with the aerodynamic heating. Also, the ability to predict the shape and location of shock waves is of primary importance in analysis of aerodynamic interference. In addition, the knowledge of the shock wave displacement is especially important in waverider vehicles¹, since these hypersonic configurations usually depend on shock wave attachment at the leading edge to achieve their high lift-to-drag ratio at high-lift coefficient.

In order to study the shock wave structure, the shape, the thickness and the detachment of the shock wave are quantified by employing the following procedure: the flow is assumed to consist of three distinct classes of molecules; those molecules from the freestream that have not been affected by the presence of the leading edge are denoted as class I molecules; those molecules that, at some time in their past history, have struck and been reflected from the body surface are denoted as class II molecules; and those molecules that have been indirectly affected by the presence of the body are defined as class III molecules. Figure 3 illustrates the definition for the molecular classes.

It is assumed that the class I molecule changes to class III molecule when it collides with either class II or class III molecule. Class I or class III molecule is progressively transformed into class II molecule when it interacts with the body surface. Also, a class II molecule remains class II regardless of subsequent collisions and interactions. Hence, the transition from class I molecules to class III molecules may represent the shock wave, and the transition from class III to class II may define the boundary layer.

A typical distribution of class III molecules along the stagnation streamline for blunt leading edges is displayed in Fig. 4 along with the definition used to determine the thickness, displacement and shape of the shock wave. In this figure, X is the distance x along the stagnation streamline (see Fig. 2), normalized by the freestream mean free path λ_∞ . f_{III} is the number of molecules for class III to the total amount of molecules inside each cell.

In a rarefied flow, the shock wave has a finite region that depends on the transport properties of the gas, and can no longer be considered as a discontinuity obeying the classical Rankine-Hugoniot relations. In this connection, the shock standoff distance Δ is defined as being the distance between the shock wave center and the nose of the leading edge along the stagnation streamline. As shown in Fig. 4, the center of the shock wave is defined by the station that corresponds to the maximum value for f_{III} . The shock wave thickness δ is defined by the distance between the stations that correspond to the mean value for f_{III} . Finally, the shock wave shape (shock wave “location”) is determined by the coordinate points given by the maximum value in the f_{III} distribution along the lines departing from the body surface, i.e., η -direction as shown in Fig. 2.

The molecule classification that has been adopted here was first presented by Lubonski²⁹ in order to study the hypervelocity Couette flow near the free molecular regime. Lubonski²⁹ divided the gas into three classes of molecules: “freestream”, “reflected from the boundary” and “scattered”. Later, for the purpose of flow visualization, Bird³⁰ applied the same scheme of classification by identifying the classes by colors: blue for class I, red for class II and yellow for class III molecules.

V. Computational Results and Discussion

The purpose of this section is to discuss and to compare differences in the shape, thickness and displacement of the shock wave due to nose-thickness and afterbody-shape variations, and to compare them to those obtained for the circular cylinder shape that generated the blunt shapes.

Figure 3. Drawing illustrating the classification of molecules.

Figure 4. Schematic of shock wave structure.

Figure 5. Distributions of molecules for classes I, II and III along the stagnation streamline for four combinations of flat-nose thicknesses and afterbody shapes: (a) $n = 2/3$ and $t/\lambda_\infty = 0$, (b) $n = 2/3$ and $t/\lambda_\infty = 1$, (c) $n = 0.8$ and $t/\lambda_\infty = 0$, e (d) $n = 0.8$ and $t/\lambda_\infty = 1$.

The distribution of molecules for the three classes along the stagnation streamline is demonstrated in Fig. 5 for four cases that combine two different nose thicknesses, t/λ_∞ of 0 and 1, with two afterbody shapes, exponent n of $2/3$ and 0.8 . The class distributions for the other combinations investigated in this work are intermediate to these four cases and, therefore, they will not be shown.

Referring to Fig. 5, f_I , f_{II} and f_{III} are the ratio of the number of molecules for class I, II and III, respectively, to the total amount of molecules inside each cell along the stagnation streamline. Of great significance in these figures is the behavior of class I molecules for sharp and blunt leading edges. It should be observed that molecules from freestream, represented by class I molecules, still collide with the nose of the leading edges as the steady state is established. This is shown in Figs. 5a and 5c, which represent sharp leading edge cases. In contrast, molecules from freestream do not reach the nose of the leading edge for cases illustrated in Figs. 5b and 5d that represent blunt leading edges. This is explained by the fact that density (not shown) increases much more for blunt (flat) leading edges in the stagnation region and reach its maximum value in the stagnation point. In this connection, the buildup of particle density near the nose of the leading edge acts as a shield for the molecules coming from the undisturbed stream.

A. Shock Wave Standoff Distance

Based on the definition shown in Fig. 4, the shock wave standoff distance Δ can be observed in Fig. 5 for the four flat-nose power-law leading edges shown.

The calculated shock wave standoff distance Δ , normalized by the freestream mean free path λ_∞ , is tabulated in Table 3 for the cases investigated. It is apparent from these results that there is a discrete shock standoff distance for the cases shown. As would be expected, the shock wave standoff distance increases with increasing the flat-nose thickness. As a reference, for power law exponent of 2/3, the shock standoff distance Δ/λ_∞ for cases t/λ_∞ of 0.01, 0.1 and 1 is, respectively, around 1.07, 1.22 and 2.15 times larger than that for the zero-thickness nose case.

Compared to flat-nose shapes, the reference circular cylinder (see Fig. 1) provides a much larger shock detachment, a Δ/λ_∞ of 1.645. Again, by taking the power law exponent of 2/3 as a reference, the shock standoff distance for the circular cylinder is about 4.98, 4.67, 4.10 and 2.32 times larger than the cases corresponding to t/λ_∞ of 0, 0.01, 0.1 and 1, respectively. The results tend to confirm the expectation that the shock standoff distance for sharp leading edge is smaller than that for blunt leading edge; it decreases with increasing the power-law exponent of the afterbody shape with the same nose thickness for the cases investigated.

It is worth mentioning that the shock standoff distance becomes important in hypersonic vehicles such as waveriders, which depend on leading edge shock attachment to achieve their high lift-to-drag ratio at high lift coefficient. In this context, the flat-nose shapes seem to be more appropriate than the circular cylinder, since they present reduced shock wave detachment distances. Nevertheless, smaller shock detachment distance is associated with a high heat load to the nose of the body. For comparison purpose, according to Santos¹³, the heat transfer coefficient C_{ho} ($\equiv 2q_w/\rho V_\infty^3$) at the stagnation point for power law exponent of 2/3 and t/λ_∞ of 0, 0.01, 0.1 and 1 is around 2.15, 2.12, 2.03 and 1.53 times, respectively, larger than that for the circular cylinder. Therefore, it should be noticed from this comparison that the ideal blunting leading edge relies on the context. If shock standoff distance is the primary issue in leading edge design of hypersonic waveriders, then flat-nose power-law leading edges are superior to round leading edges. Contrary, if the stagnation point heating is the important parameter in the hypersonic vehicle design, then round shapes seem to be superior to the flat-nose power-law shapes.

B. Shock Wave Thickness

According to the definition of the shock wave thickness shown in Fig. 4, the shock wave thickness δ along the stagnation streamline can be obtained from Fig. 5. As a result of the computation, Table 4 tabulates the shock wave thickness δ , normalized by the freestream mean free path λ_∞ , for the cases investigated.

The circular cylinder provides a larger shock thickness, δ/λ_∞ of 3.350. Compared to the flat-nose power-law shapes, this value is about 5.3, 3.9 and 2.0 times larger than the cases corresponding to t/λ_∞ of 0.01,

Table 3. Dimensionless shock wave standoff distance Δ/λ_∞ for flat-nose power-law leading edges.

n	$\frac{t}{\lambda_\infty} = 0$	$\frac{t}{\lambda_\infty} = 0.01$	$\frac{t}{\lambda_\infty} = 0.1$	$\frac{t}{\lambda_\infty} = 1$
2/3	0.330	0.352	0.401	0.708
0.7	0.289	0.312	0.358	0.676
3/4	0.216	0.237	0.312	0.654
0.8	0.166	0.187	0.271	0.640

Table 4. Dimensionless shock wave thickness δ/λ_∞ for flat-nose power-law leading edges.

n	$\frac{t}{\lambda_\infty} = 0$	$\frac{t}{\lambda_\infty} = 0.01$	$\frac{t}{\lambda_\infty} = 0.1$	$\frac{t}{\lambda_\infty} = 1$
2/3	0.887	0.904	0.978	1.575
0.7	0.790	0.831	0.894	1.520
3/4	0.646	0.722	0.794	1.449
0.8	0.523	0.619	0.694	1.403

Figure 6. Shock wave shapes on flat-nose power-law bodies with power law exponent of $3/4$ and thickness t/λ_∞ of (a) 0 , (b) 0.01 , (c) 0.1 and (d) 1 .

0.1 and 1 , respectively.

C. Shock Wave Shape

The shock wave shape, defined by the shock wave center location, is obtained by calculating the position that corresponds to the maximum f for class III molecules in the η -direction along the body surface (see Fig. 2).

Figure 6 illustrates the shock wave shape in the vicinity of the stagnation region for the flat-nose power-law bodies with thicknesses t/λ_∞ of 0 , 0.01 , 0.1 and 1 and afterbody shape defined by exponent n of $3/4$. In this set of plots, X and Y are the cartesian coordinates x and y normalized by λ_∞ .

It was pointed out by Lees and Kubota³¹ that when the freestream Mach number M_∞ is sufficiently large, the hypersonic small-disturbance equations³² admit similarity solutions for the asymptotic shock wave shapes over power-law bodies ($y \propto x^p, 0 < p < 1$), where asymptotic refers to the flowfield at large distances downstream of the nose of the body. The hypersonic small-disturbance theory states that, for certain

exponent p , a body defined by x^p produces a shock wave of similar shape and profiles of flow properties transverse to the stream direction that are similar at any axial station not too near the nose. At or near the nose, the surface slope, the curvature, and the higher derivatives are infinite, and the similarity solutions break down. In the more general case for $0 < p < 1$, the shock wave grows as x^q . When p grows from zero, q begins by keeping the constant value $q = 2/(j + 3)$, and if p keeps on growing towards unity, q remains equal to p . Here j takes the values zero for planar flow and one for axisymmetric flow.

At this point, particular attention is paid to the special case of zero-thickness with a 3/4-afterbody shape, since it is in fact a power-law shape. By considering this leading edge and by assuming that power-law bodies generate power-law shock waves in accordance with hypersonic small-disturbance theory (Lees and Kubota³¹), the shock location coordinates shown in Fig. 6a were used to approximate the shape of the shock wave with a curve fit. A fitting algorithm was performed over these points to approximate the shock shape as a power law curve of the following form,

$$y = A(x + B)^q \quad (2)$$

where A is the shock wave power law constant, B is the distance from the nose of the leading edge to the shock wave curve fit along the stagnation streamline, and q is the shock wave power law exponent.

For comparison purpose, A and B were found by keeping $q = p$, a condition for $p > 2/3$, where p and q stand for body and shock wave power law exponents, respectively. Curve fit solution for shock shape over the nose body with zero thickness and 3/4-afterbody shape is displayed in Fig. 7a. It is apparent from this figure that the curve fit matches the shock wave shape obtained by the DSMC simulation. This is in qualitative agreement with the Lees and Kubota³¹ findings in the sense that the shock wave shape would follow the shape of the body for body power law exponent $p = 3/4$.

It is worthwhile mentioning that the fitting process was performed over the points yielded by DSMC simulations located far from the nose region, say $X > 3.0$, where it is expected that the blunt nose effects are not significant. It is also important to recall that the shock wave shape in the vicinity of the nose is not correctly predicted by the theoretical solutions, since the hypersonic slender body approximations are violated close to or at the nose of the leading edges, as explained above.

The flat-nose bodies defined by Eq.(1), with y_{nose} different from 0, are not power-law shapes themselves, even though their afterbody surfaces are power-law shapes. Nevertheless, they can be closely fitted with power-law shapes ($\propto x^p$) far from the leading edge. Figure 8a depicts the comparison of the flat-nose bodies, with 3/4-afterbody shape, and the corresponding power-law curve fit shapes. As would be expected, discrepancies have been found among the curves in the vicinity of the nose of the bodies. This behavior is brought out more clearly in Fig. 8b, which exhibits details of the curves near the nose.

By considering the reference system located at the nose of the flat-nose bodies, $X = 0$, the fitting process, which has been performed over those bodies shown in Fig. 8, approximates the flat-nose body shapes by power-law shape of the following form,

$$y = C(x + D)^p \quad (3)$$

where C is the constant of the curve fit, D is the distance from the nose of the leading edge, and p is the power law exponent of the curve fit. The coefficients C and D , normalized, respectively, by λ_∞^{1-p} and λ_∞ , and the exponent p are tabulated in Table 5. The maximum absolute error between the original shapes and the curve fit shapes for $X > 3$ are less than 0.002%, 0.02% and 0.34% for thickness t/λ_∞ of 0.01, 0.1 and 1, respectively.

In what follows, the flat-nose leading edges shown in Fig. 6 are now well represented by shapes with the power-law form, $\propto x^p$, far from the nose of the leading edges. Therefore, by assuming that power-law bodies generate power-law shock waves in accordance with hypersonic small-disturbance theory (Lees and Kubota³¹), the shock location coordinates shown in Fig. 6 were used to approximate the shape of the shock wave with a curve fit as defined by Eq.(2).

Figure 7. Shock wave shapes curve fit on power-law bodies defined by $n = 3/4$ and thickness t/λ_∞ of (a) 0, (b) 0.01, (c) 0.1 and (d) 1.

Shock shape curve fit solutions for flat-nose bodies with a $3/4$ -afterbody shape and t/λ_∞ of 0.01, 0.1 and 1, which closely fits to bodies with power law exponent of 0.749, 0.746 and 0.718, respectively, are displayed from Fig. 7b to Fig. 7d. The curve fits shown in this set of figures were obtained according to Eq.(2) by three different forms; in the first form, A and B were found by keeping q equal to the body shape, $q = p$; in the second form, A , B and q were found in order to obtain the best fit; finally in the third form, A and B were found by keeping q equal to the exponent of the afterbody shape, $q = n = 3/4$.

Referring to Fig. 7, it is observed that the curve fit solutions shown present a good agreement, by visual inspection, with those solutions provided by the DSMC simulation. Nevertheless, as the maximum absolute error between the DSMC solutions and the curve fit solutions are calculated for points located at $X > 3$, it is found that the best fit is obtained for the second form of the fitting process, i.e., when A , B and q were found in order to

Table 5. Dimensionless coefficients C , D and p for the curve fit power-law bodies.

t/λ_∞	C	D	p
0.01	0.51664	0.01225	0.749
0.1	0.52301	0.12215	0.746
1	0.58549	1.24258	0.718

Figure 8. Comparison of flat-nose bodies with a 3/4-afterbody shape and thickness t/λ_∞ of 0.01, 0.1 and 1, with power-law curve fit shapes. (a) total length L , and (b) in the vicinity of the nose.

yield the best solution. In this respect, the best agreement is found for exponents q of 0.80, 0.80 and 0.77 for t/λ_∞ of 0.01, 0.1 and 1, respectively. Consequently, the computational results indicate that the shock wave shapes grow with a power-law form ($\propto x^q$), but with exponents slightly different from those of the body shapes for the flat-nose bodies with a 3/4-afterbody shape. Nonetheless, it should be emphasized that the curve fit exponents are very sensitive to the numbers of coordinate points, which define the shock wave, used in the fitting process. In addition, these coordinate points present fluctuations, originated from the DSMC simulations, which were not taken into account.

For comparison purpose, the coefficients A and B , normalized, respectively, by λ_∞^{1-q} and λ_∞ , and the exponent q are tabulated in Table 6.

Table 6. Dimensionless coefficients A , B and q for the shock wave curve fit.

t/λ_∞	0.01	0.01	0.01	0.1	0.1	0.1	1	1	1
q	0.749	0.75	0.804	0.746	0.75	0.804	0.718	0.75	0.77
A	1.19479	1.11907	0.98728	1.20949	1.19313	0.98874	1.38002	1.23552	1.15270
B	-0.44771	-0.43586	0.22898	-0.38974	-0.34183	0.33197	0.50621	0.94082	1.21609

VI. Concluding Remarks

This study applies the Direct Simulation Monte Carlo method in order to investigate the shock wave structure of flat-nose power-law leading edges. The calculations have provided information concerning the nature of the shock wave detachment distance, shock wave thickness and shock wave shape resulting from variations on the flat-nose thickness and on the afterbody shape for the idealized situation of two-dimensional hypersonic rarefied flow. The emphasis of the investigation was also to compare these flat-nose power-law leading edges with round shape in order to determine which geometry is better suited as a blunting profiles in terms of the shock wave standoff distance.

It was found that the shock wave standoff distance and the shock wave thickness for the flat-nose power-law bodies are smaller than that for the circular body with the same tangency to a wedge of specified oblique angle. In this context, the flat-nose power-law leading edges seem to be more appropriate than the circular cylinder, since they present reduced shock wave detachment distances, an important property for hypersonic waveriders. In addition, the computational results indicated that the shock wave shape grows with power-law form ($\propto x^q$) for the flat-nose bodies investigated, which can be closely fitted with power-law shapes ($\propto x^p$).

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