A&A 390, 351–357 (2002)
DOI: 10.1051/0004-6361:20020693

Astronomy & Astrophysics

A theory of the fundamental plasma emission of type-III solar radio bursts

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Received 25 March 2002 / Accepted 6 May 2002

Abstract. Results from plasma wave experiments in spacecraft give support to nonlinear interactions involving Langmuir waves, electromagnetic waves and ion-acoustic waves in association with type III solar radio bursts. In this paper we present a theory of the fundamental plasma emission of type-III solar radio bursts. Starting from the generalized Zakharov equations, considering the pump wave as a pair of oppositely propagating Langmuir waves with different amplitudes, and the excitation of electromagnetic and induced Langmuir waves, we obtain a general dispersion relation for the coupled waves. We numerically solve the general dispersion relation using the pump wave amplitude and plasma parameters as observed in the interplanetary medium. We compare our results with previous models. We find that the stability properties depend on the pump wave numbers and on the ratio of wave amplitude between the forward and backward pump wave. The inclusion of a second pump wave allows the simultaneous generation of up and down converted electromagnetic waves. The presence of a second pump with different amplitude from the first one brings a region of convective instability not present when amplitudes are the same.

Key words. instabilities – plasmas – waves – radio radiation – solar wind

1. Introduction

According to the presently accepted ideas, type III solar radio bursts are produced by a two-step process: first, an energetic electron beam excites electrostatic oscillations, Langmuir waves, at the fundamental plasma frequency $f_{\rm p}$, and then the Langmuir waves are converted into electromagnetic radiation via non-linear wave-wave interactions.

Several wave-wave interaction models have been proposed to explain the second step that generates the electromagnetic radiation. In a classical paper, Ginzburg & Zhelezyakov (1958) proposed that an electromagnetic wave at the fundamental plasma frequency could be produced by a three-wave process, involving a nonlinear coupling of a Langmuir pump wave and an ion-acoustic daughter wave. Akimoto (1988) and Abalde et al. (1998) extended the three-wave model of Ginzburg & Zhelezyakov (1958) to a four-wave model based on a modulational hybrid (electromagnetic-electrostatic) instability in which either Stokes or anti-Stokes electromagnetic and Langmuir daughter waves are simultaneously excited.

Lashmore-Davies (1974) was the first to show that the fundamental plasma emission can result from the nonlinear coupling of two wave triplets due to counterpropagating Langmuir waves. A theory of type III solar radio-bursts was formulated by Chian & Alves (1988), based on the idea of Lashmore-Davies (1974), for the case of two-pump Langmuir waves with the same wave amplitudes. In the models of Lashmore-Davies (1974) and Chian & Alves (1988) only one grating (ion-acoustic mode) was included. Rizzato & Chian (1992) improved their models by self-consistently including a second grating that assures the symmetry of the wave kinematics. Moreover, Rizzato & Chian (1992) investigated the simultaneous generation of electromagnetic and Langmuir daughter waves, thus we treat their model as a two-pump version of the hybrid instability studied by Akimoto (1988) and Abalde et al. (1998). Glanz et al. (1993) modified the model of Chian & Alves (1988) to allow for different amplitudes for the two-pump Langmuir waves and included a second grating.

The aim of this paper is to generalize the model of Rizzato & Chian (1992) to the case of two oppositely propagating Langmuir pump waves with distinct wave amplitudes. This is accomplished by deriving a general dispersion relation based on the generalized Zakharov equations. The present paper may also be regarded as a generalization of the model of Glanz et al. (1993) to include the excitation of Langmuir daughter waves.

This theory will provide a better framework for understanding the observed features of type III solar radio bursts which often indicate the signature of two populations of Lagmuir waves

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traveling in opposite directions. In real situations, the two counterstreaming type III Lagmuir waves are expected to have different amplitudes.

This paper is organized as follows: in Sect. 2 we derive the general dispersion relation. In Sect. 3 we present the previous one-pump models and in Sect. 4 the previous two-pump models. In Sect. 5 we discuss the results and Sect. 6 presents the conclusions.

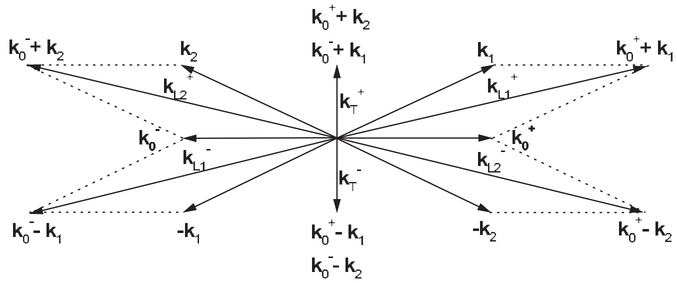

Fig. 1. Wave-vector knematics for our model: $k_0^{+(-)}$ are related to the pump Langmuir waves, $k_{1(2)}$ to ion acoustic waves, $k_0^{+(-)} - k_{1(2)}$ are the electrostatic (oblique to $k_0^{+(-)}$) or electromagnetic ($\perp$ to $k_0^{+(-)}$) Stokes modes and $k_0^{+(-)} + k_{1(2)}$ are the electrostatic (oblique to $k_0^{+(-)}$) or electromagnetic ($\perp$ to $k_0^{+(-)}$) anti-Stokes modes.

2. Derivation of the general dispersion relation

The nonlinear coupling of Langmuir waves (L), electromagnetic waves (T) and ion-acoustic waves (S) is governed by the generalized Zakharov equations (Akimoto 1988; Rizzato & Chian 1992; Chian & Abalde 1997; Robinson 1997; Abalde et al. 1998; Bárta & Karlicky 2000)

$$\left(\partial_t^2 - \nu_e\partial_t + c^2\nabla\times(\nabla\times) - \gamma_e v_{th}^2\nabla(\nabla\cdot) + \omega_p^2\right)\vec{E} = -\frac{\omega_p^2}{n_0}n\vec{E}, \quad (1)$$

$$\left(\partial_t^2 - \nu_i\partial_t - v_s^2\nabla^2\right)n = \frac{\varepsilon_0}{2m_i}\nabla^2 < \vec{E}^2 > , \quad (2)$$

where $\vec{E}$ is the high-frequency electric field, n is the ion density fluctuation, $\omega_p^2 = n_0 e^2/(m_e\varepsilon_0)$ is the electron plasma frequency, c is the velocity of light, $v_{th} = (k_B T_e/m_e)^{1/2}$ is the electron thermal velocity, $v_s = (k_B(\gamma_e T_e + \gamma_i T_i)/m_i)^{1/2}$ is the ion-acoustic velocity, $\nu_{e(i)}$ is the damping frequency for electrons (ions), $\gamma_{e(i)}$ is the ratio of the specific heats for electrons (ions), and the angle brackets denote the fast time average.

In order to derive a dispersion relation from Eqs. (1) and (2) we assume that the electric field of the Langmuir pump wave is given in the form

$$\vec{E}_0 = \frac{1}{2}\left(\vec{E}_0^+\exp(i(\vec{k}_0\cdot\vec{r} - \omega_0 t)) + \vec{E}_0^-\exp(i(-\vec{k}_0\cdot\vec{r} - \omega_0 t))\right) + c.c.,$$

which represents two oppositely propagating waves of equal frequencies and opposite wave vectors, the amplitude of two pump waves can be different, $\left|\vec{E}_0^-\right|^2 = r\left|\vec{E}_0^+\right|^2$, with $0 \le r \le 1$. Each of the two pump waves can generate the Stokes ($\omega_0 - \omega^*$) and the anti-Stokes ($\omega_0 + \omega$) modes, either electrostatic or electromagnetic, as known in three-wave processes. We consider the coupling of four triplets. Each two triplets have in common the Langmuir pump wave (forward or backward wave) and a pair of independent density gratings, $n_{1(2)}$, given by

$$n = \frac{1}{2}n_{1(2)}\exp\left(i(\vec{k}_{1(2)}\cdot\vec{r} - \omega t)\right) + c.c.,$$

with $\left|\vec{k}_1\right| = \left|\vec{k}_2\right|$. The electric fields of the daughter waves obey the relation

$$\vec{E} = \frac{1}{2}\left(\vec{E}_w^{+(-)}\exp(i(\vec{k}_w^{+(-)}\cdot\vec{r} - \omega_w^{+(-)}t))\right) + c.c.,$$

where the subscript w represents either electromagnetic (T) or Langmuir (L) daughter wave, respectively; the superscript +(−) represents the anti-Stokes (Stokes) mode, respectively. Figure 1 illustrates this coupling and also shows the wave-vector matching conditions we have assumed.

The assumption of two oppositely propagating pumps implies the need of some process to cause the E_0^+-waves to be scattered into the backward direction. High time resolution electric field waveform measurements from the Galileo spacecraft have shown that Langmuir waves associated with type III solar radio bursts have a characteristic beat pattern (Gurnett et al. 1993). This beat pattern is produced by two closely spaced narrowband components. One of these components is believed to be the beam-driven Langmuir wave, E_0^+ , and the other is believed to be the oppositely propagating Langmuir wave generated by the parametric decay of E_0^+. For the sake of simplicity, we assume in this paper that the two pump waves are anti-parallel. It is worth to point out that the Langmuir decay process can generate a spectrum of $\vec{k}$ not collinear with $\vec{k}_0$. This difference can influence the results and should be studied in the future.

The dispersion relation, considering the wave number and frequency matching conditions, can be obtained using either the propagator technique or the Fourier transform technique of the assumed electric fields. It has been shown that these two techniques lead to the same results (Rizzato & Chian 1992).

The chosen kinematics implies that the two Langmuir pumps propagate oppositely along the longitudinal x-axis, generating two opposite induced electromagnetic modes that primarily propagate along transverse y-axis, plus induced Langmuir modes that mainly propagate along the x-axis. Writing the total high-frequency fluctuating field in terms of its transverse and longitudinal component, $\vec{E} = \vec{E}_L + \vec{E}_T$, imposing perfect $\vec{k}$-matching but allowing for frequency mismatches between the interacting waves and using the kinematic conditions presented in Fig. 1 in Eq. (1), we can write the variations of the induced waves as follows

$$\mathcal{D}_{L1}^- E_{L1}^- = \frac{\omega_p^2}{n_0}n_1^* E_0^- , \; \mathcal{D}_{L1}^+ E_{L1}^+ = \frac{\omega_p^2}{n_0}n_1 E_0^+, \quad (3)$$

$$\mathcal{D}_{L2}^- E_{L2}^- = \frac{\omega_p^2}{n_0}n_2^* E_0^+, \mathcal{D}_{L2}^+ E_{L2}^+ = \frac{\omega_p^2}{n_0}n_2 E_0^-, \quad (4)$$

$$\mathcal{D}_T^+ E_T^+ = \frac{\omega_p^2}{n_0}\left(n_1 E_0^- + n_2 E_0^+\right), \text{ and}$$

$$\mathcal{D}_T^- E_T^- = \frac{\omega_p^2}{n_0}\left(n_1^* E_0^+ + n_2^* E_0^-\right). \tag{5}$$

In Eqs. (3–5) sub-indexes L1(2) refer to the Langmuir wave daughter due do the first (second) grating.

Introducing Eqs. (3–5) in Eq. (2), we obtain the following equations for the density fluctuations

$$\begin{aligned}\mathcal{D}_{s1} n_1 &= \frac{\varepsilon_0 k_1^2 \omega_p^2}{2 m_i n_0}\left(\frac{n_1 \left|E_0^+\right|^2}{\mathcal{D}_{L1}^+} + \frac{n_1 \left|E_0^-\right|^2}{\mathcal{D}_{L1}^{-*}} + \frac{n_1 \left|E_0^-\right|^2}{\mathcal{D}_T^+}\right.\\ &\left. + \frac{n_2 E_0^+ E_0^{-*}}{\mathcal{D}_T^+} + \frac{n_1 \left|E_0^+\right|^2}{\mathcal{D}_T^{-*}} + \frac{n_2 E_0^+ E_0^{-*}}{\mathcal{D}_T^{-*}}\right),\end{aligned} \tag{6}$$

$$\begin{aligned}\mathcal{D}_{s2} n_2 &= \frac{\varepsilon_0 k_2^2 \omega_p^2}{2 m_i n_0}\left(\frac{n_2 \left|E_0^+\right|^2}{\mathcal{D}_{L2}^{-*}} + \frac{n_2 \left|E_0^-\right|^2}{\mathcal{D}_{L2}^+} + \frac{n_2 \left|E_0^+\right|^2}{\mathcal{D}_T^+}\right.\\ &\left. + \frac{n_1 E_0^{+*} E_0^-}{\mathcal{D}_T^+} + \frac{n_2 \left|E_0^-\right|^2}{\mathcal{D}_T^{-*}} + \frac{n_1 E_0^{+*} E_0^-}{\mathcal{D}_T^{-*}}\right).\end{aligned} \tag{7}$$

In Eqs. (6) and (7) we have:

$$\begin{aligned}\mathcal{D}_{s1(2)} &= \omega^2 + i\nu_i\omega - v_s^2 k_{1(2)}^2,\\ \mathcal{D}_T^\pm &= \left((\omega_o \pm \omega)^2 + i\nu_T(\omega_o \pm \omega) - c^2 k_T^{\pm 2} - \omega_p^2\right),\\ \mathcal{D}_{L1(2)}^\pm &= \left((\omega_o \pm \omega)^2 + i\nu_L(\omega_o \pm \omega) - v_{th}^2 k_{L1(2)}^{\pm 2} - \omega_p^2\right).\end{aligned}$$

Observe that $\left|\vec{k}_{L1}^{\pm}\right| = \left|\vec{k}_{L2}^{\pm}\right|$, and $\left|\vec{k}_T^{+}\right| = \left|\vec{k}_T^{-}\right|$, as shown in Fig. 1.

Using the high-frequency approximation $\mathcal{D}_T^\pm \cong \pm 2\omega_p(\omega \pm (\omega_0 - \omega_T) + i\nu_T/2)$, $\mathcal{D}_{L1(2)}^\pm \cong \pm 2\omega_p(\omega \pm (\omega_0 - \omega_L) + i\nu_L/2)$ with $\omega_{L(T)}$ representing the linear relation for the Langmuir (electromagnetic) wave, $\left|\vec{k}_1\right| = \left|\vec{k}_2\right|$, and normalizing ω by ω_s and k by $k\lambda_D$ we introduce

$$\begin{aligned}D_s &= \left(\omega^2 - 1 + i\nu_i/2\right)\\ D_T^\pm &= \omega \pm \frac{3}{2}\frac{k_0}{(\mu\tau)^{1/2}} \mp \frac{1}{2}\frac{c^2}{v_{th}^2}\frac{k_T^2}{(\mu\tau)^{1/2} k_0}\\ D_L^\pm &= \omega \mp \frac{9}{2}\frac{k_0}{(\mu\tau)^{1/2}}\end{aligned}$$

where $\mu = m_e/m_i$, and $\tau = (\gamma_e T_e + \gamma_i T_i)/T_e$. Finally, assuming $\left|\vec{E}_0^-\right|^2 = r\left|\vec{E}_0^+\right|^2$, and defining $W_0 = \varepsilon_0\left|\vec{E}_0^+\right|^2/(2n_0 k_B T_e)$, and $W_{T0} = (1+r)W_0$, we obtain the general dispersion relation

$$\begin{aligned}D_s^2 &- \frac{W_{T0}}{4\tau(\mu\tau)^{1/2} k_0} D_s\left(\frac{1}{D_L^+} - \frac{1}{D_L^-} + \frac{1}{D_T^+} - \frac{1}{D_T^-}\right)\\ &+ \frac{W_{T0}^2}{16\mu\tau^3 k_0^2(1+r)^2}\left[\frac{-(1-r)^2}{D_T^+ D_T^-}\right.\\ &+ (1+r^2)\left(\frac{1}{D_T^+ D_L^+} - \frac{1}{D_L^+ D_L^-} + \frac{1}{D_T^- D_L^-}\right)\\ &\left. - 2r\left(\frac{1}{D_L^+ D_T^-} + \frac{1}{D_T^+ D_L^-}\right) + r\left(\frac{1}{D_L^{+2}} + \frac{1}{D_L^{-2}}\right)\right] = 0.\end{aligned} \tag{8}$$

In the next section, we present the dispersion relations and wave vector kinematics used in previous models for the wave-wave interaction that generates the electromagnetic radiation in solar type III radio bursts and their relation with Eq. (8).

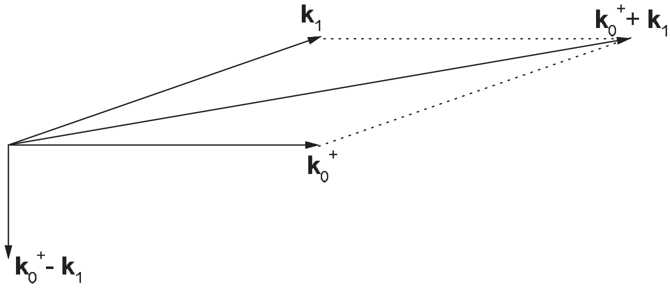

Fig. 2. Wave-vector matching conditions for the Akimoto (1988) model; k_0^+ is the wave vector of the pump wave, k_1 of the ion acoustic wave, $k_0^+ - k_1$ of the Stokes mode (electromagnetic daughter wave) and $k_0^+ + k_1$ of the anti-Stokes mode (Langmuir daughter wave).

3. Comparison with the previous one-pump models

Various mechanisms have been suggested for generating the solar fundamental radiation considering the pump as a traveling Langmuir wave. Traditionally, type III events are interpreted in terms of a three-wave process (Ginzburg & Zheleznyakov 1958). However, some recent studies indicated that hybrid parametric instabilities, involving the nonlinear coupling of two or more wave triplets are usually produced by a Langmuir pump wave.

Akimoto (1988) treated the case where the one-pump Langmuir wave can decay into a Langmuir wave and an electromagnetic wave, via $L \longrightarrow L^+ + T^- + S$. The wave vector geometry he considered, with $\vec{k}_o^+ \approx \vec{k}_1$ is shown in Fig. 2. In his model, the excited electromagnetic wave is the Stokes mode, with frequency given by $\omega_0 - \omega$ and wave vector $\vec{k}_T = \vec{k}_o^+ - \vec{k}_1$ with $\left|\vec{k}_T\right| < \left|\vec{k}_o^+\right|, \left|\vec{k}_1\right|$. The Langmuir daughter wave is the anti-Stokes mode, with frequency given by $\omega_0 + \omega$ and wave vector $\vec{k}_T = \vec{k}_o^+ + \vec{k}$.

The dispersion relation obtained by Akimoto (1988) is given by:

$$D_s - \frac{W_0}{4\tau(\mu\tau)^{1/2} k_0}\left(\frac{1}{D_L^+} - \frac{1}{D_T^-}\right) = 0. \tag{9}$$

Observe that Eq. (9) can be obtained from Eq. (8) considering $r = 0$, the presence of only one density grating ($\vec{k}_1 \approx \vec{k}_0^+$), and assuming that only the modes T^- and $L^+$are excited.

Abalde et al. (1998) treated a similar process, where a Langmuir pump wave can also generate a Langmuir wave and an electromagnetic wave through $L \longrightarrow L^- + T^+ + S$. The difference here is $\vec{k}_0^+ \approx -\vec{k}_2$, and as a consequence, the excited electromagnetic wave is the anti-Stokes mode with frequency $\omega_0 + \omega$ and wave vector $\vec{k}_T = \vec{k}_0^+ + \vec{k}_2$, and the Langmuir daughter wave is the Stoke mode with frequency $\omega_0 - \omega$ and wave vector $\vec{k}_0^+ - \vec{k}_2$. The wave vector kinematic for this process is shown in Fig. 3. In their model it was also assumed that $\left|\vec{k}_T\right| < \left|\vec{k}_o^+\right|, \left|\vec{k}_2\right|$.

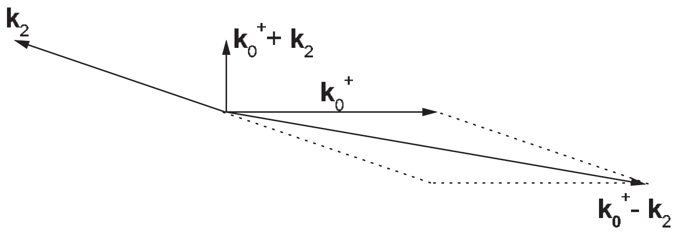

Fig. 3. Wave-vector matching conditions for the Abalde et al. (1998) model; k_0^+ is the wave vector of the pump wave, k_2 of the ion-acoustic wave, $k_0^+ - k_2$ of the Stokes mode (Langmuir daughter wave) and $k_0^+ + k_2$ of the anti-Stokes mode (electromagnetic daughter wave).

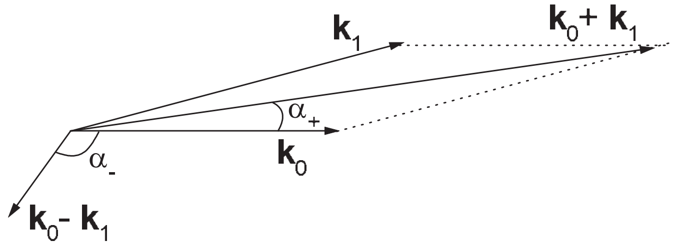

Fig. 4. Wave-vector matching conditions for the Bárta & Karlicky (2000) model; k_0^+ is the wave vector of the pump wave, k_1 of the ion-acoustic wave, $k_0^+ - k_1$ of the Stokes mode and $k_0^+ + k_1$ of the anti-Stokes mode.

The dispersion relation obtained by Abalde et al. (1998) is given by:

$$D_\mathrm{s} - \frac{W_0}{4\tau(\mu\tau)^{1/2}k_0}\left(\frac{1}{D_\mathrm{T}^+} - \frac{1}{D_\mathrm{L}^-}\right) = 0. \tag{10}$$

Observe that Eq. (10) can be obtained from Eq. (8) considering $r = 0$, the presence of only one density grating ($\vec{k}_2 \approx -\vec{k}_0^+$), and assuming that only the modes T^+ and L^- and are excited.

More recently, Bárta & Karlicky (2000) derived a dispersion relation for hybrid processes with one Langmuir pump wave. They included an angular dependence, previously introduced by Akimoto (1988), represented by $\alpha_\pm$ in Fig. 4, which shows the wave vector kinematics for their process. Observe that depending on the value of $\alpha_\pm$, the angle between $\vec{k}_0^+$ and $\vec{k}_0^+ \pm \vec{k}_1$, we can have the model of Akimoto (1988), or Abalde et al. (1998). In Bárta & Karlicky (2000) model there is no restrictions a priori about the magnitude of the wave vectors.

4. Comparison with the previous two-pump models

In a pioneer work, Lashmore-Davies (1974) demonstrated that an efficient process for converting intense Langmuir waves into fundamental electromagnetic waves could be achieved by two counterstreaming Langmuir pump waves.

Chian & Alves (1988) considered the process induced by two counterstreaming Langmuir pump waves with equal amplitudes ($r = 1$) including both decay-type ($L \longrightarrow T + S$) and fusion-type ($L + S \longrightarrow T$). The kinematics of wave-vector they used is shown in Fig. 5.

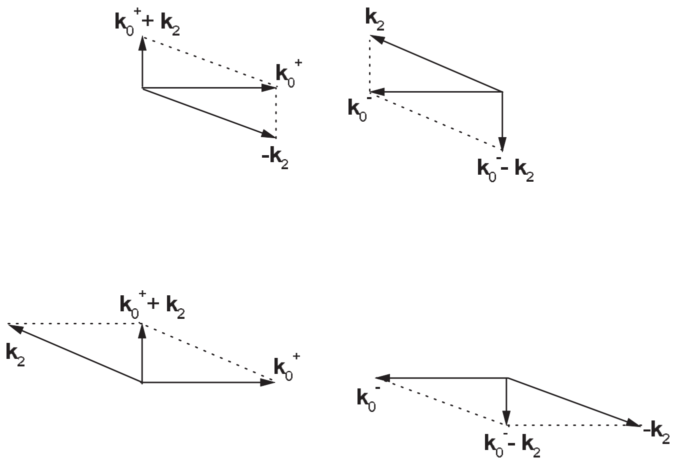

Fig. 5. Wave-vector matching conditions for the Chian & Alves (1988) model; $k_0^\pm$ is the wave vector of the pump waves, k_2 of the ion-acoustic wave, $k_0^\pm - k_2$ of the Stokes mode and $k_0^\pm + k_2$ of the anti-Stokes mode, both electromagnetic waves.

Chian & Alves (1988) obtained the following dispersion relation:

$$D_\mathrm{s} - \frac{W_0}{2\tau(\mu\tau)^{1/2}k_0}\left(\frac{1}{D_\mathrm{T}^+} - \frac{1}{D_\mathrm{T}^-}\right) = 0. \tag{11}$$

Observe that Eq. (11) can be obtained from Eq. (8) by setting $r = 1$, considering one density grating, $\vec{k}_2$, and only the excitation of electromagnetic waves $T^\pm$.

Rizzato & Chian (1992) adopted the generalized Zakharov equations to re-examine the decay-type electromagnetic parametric instabilities driven by counterpropagating Langmuir waves of equal amplitudes. They considered that the instability excites both high-frequency electromagnetic and Langmuir waves. They used the same wave-vector matching conditions we presented in Fig. 1. The dispersion relation they obtained is given by:

$$D_\mathrm{s} - \frac{W_0}{4\tau(\mu\tau)^{1/2}k_0}\left[2\left(\frac{1}{D_\mathrm{T}^+} - \frac{1}{D_\mathrm{T}^-}\right) + \left(\frac{1}{D_\mathrm{L}^+} - \frac{1}{D_\mathrm{L}^-}\right)\right] = 0. \tag{12}$$

In that case, we should expect that by setting $r = 1$ in Eq. (8) we would recover Eq. (12). In reality, we observe that this is not the case. It is important to point out that the assumption of $r = 1$ in Eqs. (6) and (7) allows us to write two uncoupled equations in terms of two independent solutions, $(n_1 + n_2)$, purely electrostatic modes, and $(n_1 - n_2)$, hybrid modes (electrostatic and electromagnetic). Equation (12) is obtained from the equation for $(n_1 - n_2)$.

Glanz et al. (1993) investigated the electromagnetic instability from two counterpropagating Langmuir waves with different amplitudes. They did not consider the presence of the Langmuir daughter waves, thus they used the wave-vector matching conditions presented in Fig. 6.

Glanz et al. (1993) obtained the following dispersion relation:

$$D_\mathrm{s}^2 - \frac{W_{\mathrm{T}0}}{4\tau(\mu\tau)^{1/2}k_0}D_\mathrm{s}\left(\frac{1}{D_\mathrm{T}^+} - \frac{1}{D_\mathrm{T}^-}\right) - \frac{W_{\mathrm{T}0}^2}{16\mu\tau^3k_0^2}\left(\frac{(1-r)}{(1+r)}\right)^2\frac{1}{D_\mathrm{T}^+D_\mathrm{T}^-} = 0. \tag{13}$$

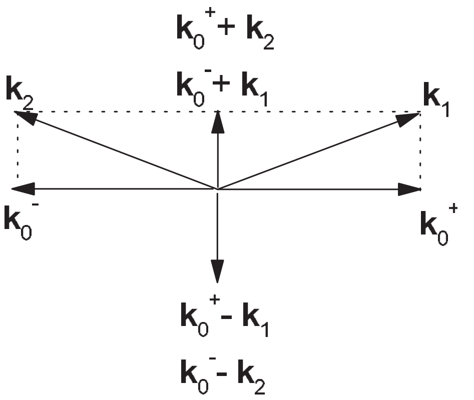

Fig. 6. Wave-vector matching conditions for the Glanz et al. (1993) model; $k_0^{\pm}$ is the wave vector of the pump waves, $k_{1(2)}$ of the ion acoustic wave, $k_0^{\pm} - k_{1(2)}$ of the Stokes mode and $k_0^{\pm} + k_{1(2)}$ of the anti-Stokes mode, both electromagnetic waves.

Notice that Eq. (13) reduces to Eq. (11), Chian & Alves (1988) model, for $r = 1$ ($W_{\rm T0} = 2W_0$). Moreover, Eq. (13) is recovered from Eq. (8) when we consider that only electromagnetic waves, $T^{\pm}$, are excited, and $r = 1$.

5. Discussions

In order to compare the several models we discussed in the previous sessions we present next the numerical solutions for some of the dispersion relations. Numerical results are obtained using parameters relevant to type III solar radio bursts (Thejappa & MacDowall 1998). We use a fixed value of $k_0 = 0.0451$, $W_0 = 10^{-5}$, $\mu = 1/1836$, $v_{\rm th} = 2.2 \times 10^6$ m/s, $T_{\rm e} = 1.6 \times 10^5$ K, and $T_{\rm i} = 5 \times 10^4$ K. For the sake of simplicity, we set the damping frequencies to zero in our computations.

It has been shown that a three-wave electromagnetic instability, with the ion-acoustic wave as a resonant mode ($Re\{\omega\} \approx \omega_{\rm s}$), in the case of one-pump wave, takes place when $k_0 > (2/3)\,(\mu\tau)^{1/2}$ (Akimoto 1988; Abalde et al. 1998). For the experimental parameters, we note that this condition is satisfied.

Using the parameters above we find the numerical solutions for the Akimoto (1988) model, Eq. (9) and for the Abalde et al. (1998) model, Eq. (10). Figure 7 presents the results. Figures 7a and 7b present the real part of ω, $\omega_{\rm r}$ and Figs. 7c and 7d the imaginary part (or growth rates) of ω, denoted by Γ, as a function of $k_{\rm T}\lambda_{\rm D}$. We see that instability occurs for the same interval of $k_{\rm T}\lambda_{\rm D}$ and has the same growth rate for both models. Notice that the real part of the frequency is >0 for the Akimoto (1988) model, (a), and <0 for the Abalde et al. (1998) model, (b). However, as in the Abalde et al. (1998) model the frequency of the electromagnetic wave is given by $\omega_0 + \omega_{\rm r}$ (anti-Stokes mode) and in the Akimoto (1988) model by $\omega_0 - \omega_{\rm r}$ (Stokes-mode), the final results are the same. Both leads to down converted electromagnetic waves. This fact has already been pointed out by Bárta & Karlicky (2000).

It was also shown that for one-pump model, within the limit $k_0 < (1/3)W_0^{1/2}$, the ion- acoustic wave is a non-resonant mode ($\mathrm{Re}\{\omega\} \neq \omega_{\rm s}$). In order to investigate this we find numerical solutions for Eqs. (9) and (10), for $k_0 = 10^{-4}$, keeping all the other parameters the same. Figure 8 presents the

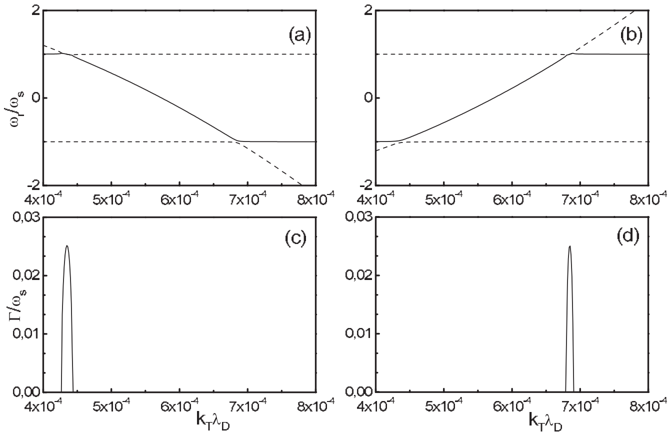

Fig. 7. Numerical solutions for one-pump models within the limit $k_0 > (2/3)\,(\mu\tau)^{1/2}$; **a)** and **c)** for the Akimoto (1988) model, and **b)** and **d)** for the Abalde et al. (1998) model; **a)** and **b)** plot the real part of the solution and **c)** and **d)** the growth rate, as a function of $k_{\rm T}\lambda_{\rm D}$.

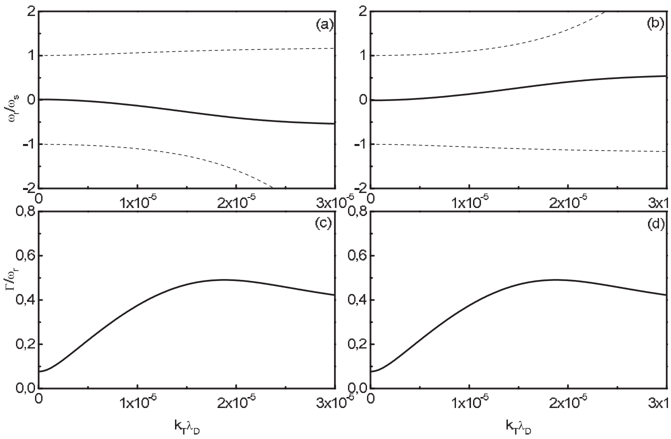

Fig. 8. Numerical solution of equation for the Akimoto (1988) (**a)** and **c)**) and Abalde et al. (1998) (**b)** and **d)**) models, within the limit $k_0 < (1/3)W_0^{1/2}$; **a)** and **b)** show the real frequencies, and **c)** and **d)** the growth rates.

solutions. Figures 8a and 8b presents the real frequency and Figs. 8b and 8d the growth rates. Observe that within this limit we have nearly purely growing instability with $\omega_{\rm r} \leq 0$ for the Akimoto (1988) model ((a) and (c)), and with $\omega_{\rm r} \geq 0$ for the Abalde et al. (1998) model ((b) and (d)), again showing that both models lead to the same result. Within this limit we can obtain up-converted electromagnetic waves for high values of $k_{\rm T}\lambda_{\rm D}$, as shown in Fig. 8.

We now discuss the two-pump models. Using the observational parameters, $k_0 = 0.0451$, $W_0 = 10^{-5}$, $\mu = 1/1836$, $v_{\rm th} = 2.2 \times 10^6$ m/s, $T_{\rm e} = 1.6 \times 10^5$ K, and $T_{\rm i} = 5 \times 10^4$ K (Thejappa & MacDowall 1998) and $r = 1$, we solved the general dispersion relation, Eq. (8), derived in this paper and the equation for the Rizzato & Chian (1992) model, Eq. (12). Figure 9 presents the solution for Eq. (8), which is the same as the solution of Eq. (12). The real part of the solution, Fig. 9a, encompass the Akimoto (1988) and the Abalde et al. (1998) models. The growth rate, Fig. 9b, is similar to both models. The inclusion of the second pump wave allows the simultaneous generation of up and down converted electromagnetic waves.

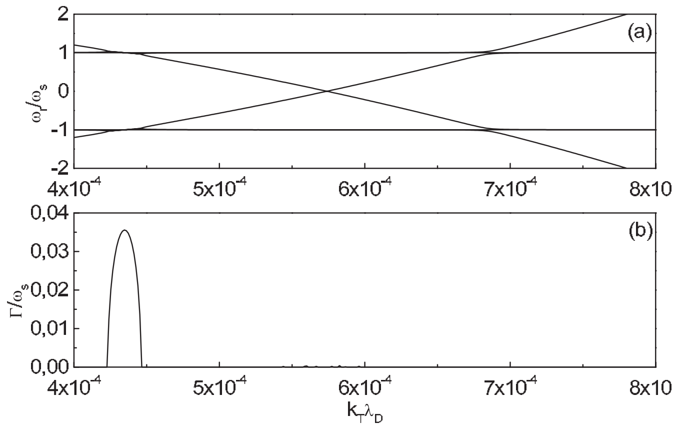

Fig. 9. Numerical solutions for the Rizzato & Chian (1992) and the present model within the limit $k_0 > (2/3)\,(\mu\tau)^{1/2}$; **a)** shows the real part of the solution and **b)** growth rate.

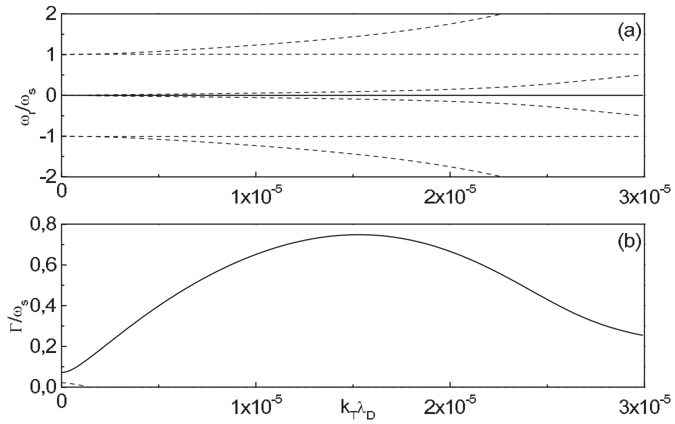

Fig. 10. Numerical solutions for the Rizzato & Chian (1992) and the present model; **a)** shows the real part of the solution and **b)** growth rate, within the limit $k_0 < (1/3)W_0^{1/2}$, with $k_0 = 10^{-4}$.

Up converted electromagnetic waves can readily propagate in the surrounding regions of slightly higher plasma density without being absorbed.

Since Eq. (8) is not trivial to treat analytically, even to look for purely growing solutions, we find solutions for a different set of parameters, basically changing the value of k_0. We use the same set of parameters used to check the limit $k_0 < (1/3)W_0^{1/2}$ for the one-pump wave model, looking for solutions where the ion-acoustic wave is a non-resonant mode ($\omega_r \neq \omega_s$).

Figure 10 presents the solutions for the Rizzato & Chian (1992) and the present model, Eq. (8), for $k_0 = 10^{-4}$. Solutions for both models are the same except by the solution $\omega_r/\omega_s = 1$, not present in the Rizzato & Chian (1992) model. This is due to the fact that in the Rizatto & Chian (1992) model it was possible to decouple pure electrostatic modes, $(n_1 + n_2)$, from the hybrid ones, $(n_1 - n_2)$. Equation (12) gives the solution only for the hybrid modes. Figure 10a shows the real part of the solution and Fig. 10b the growth rates. All the dashed lines are real solutions. Instability within the limit $k_0 < (1/3)W_0^{1/2}$ is purely growing, different from the one-pump models (see Fig. 8).

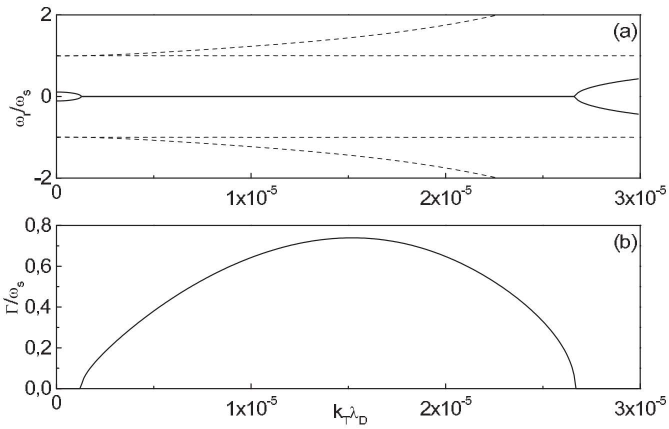

Fig. 11. Numerical solutions for the Chian & Alves (1988) or Glanz et al. (1993) model; **a)** shows the real part of the solution and **b)** growth rate, within the limit $k_0 < (1/3)W_0^{1/2}$, with $k_0 = 10^{-4}$.

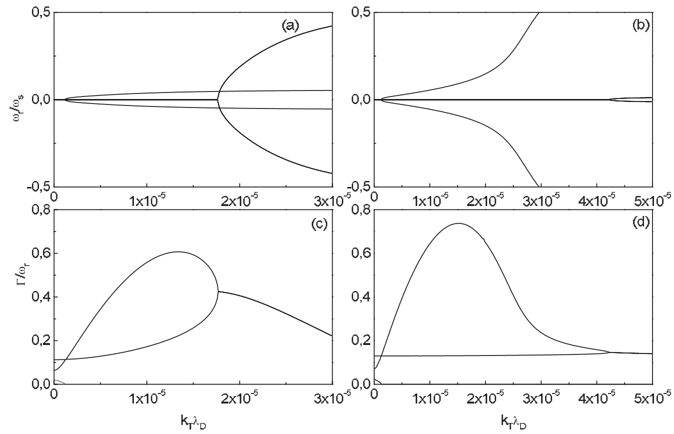

Fig. 12. Numerical solutions for the present model with different values of r; **a)** and **b)** show the real part of the solution and **b)** and **d)** the growth rates, with $k_0 = 10^{-4}$, within the limit $k_0 < (1/3)W_0^{1/2}$; **a)** and **c)** refers to $r = 0.5$, and **b)** and **d)** to $r = 0.95$.

The inclusion of Langmuir wave daughters leads to a broadening in the $k_T\lambda_D$ region of instability as compared to the Chian & Alves (1988) or the Glanz et al. (1993) model that only includes excited electromagnetic waves. Solution of Eqs. (11) or (13) for the same parameters used in Fig. 10 is presented in Fig. 11, that shows a limited range for a purely growing instability. All the dashed lines are real solutions.

In order to verify the relative importance of the second wave pump amplitude, we solve Eq. (8) for different values of r. Figure 12 shows the results for $r = 0.5$ ((a) and (c)), and for $r = 0.95$ (b) and (d)). The presence of a second pump wave with different amplitude from the first one introduces a region of convective instability ($\omega_r \neq 0$), not present when $r = 1$ (see Fig. 10). For small values of $k_T\lambda_D$, the instability is purely growing, $\omega_r = 0$. For a critical value of $k_T\lambda_D$ the real part of the frequency bifurcates off the line $\omega_r = 0$, while still presenting an imaginary part $\Gamma \neq 0$. The critical value of $k_T\lambda_D$ decreases as r decreases.

6. Conclusions

In conclusion, we derived a general dispersion relation for the parametric generation of fundamental plasma emissions due to two counterstreaming Langmuir waves with different wave amplitudes. We performed a comparison among several models present in the literature. We observed that the inclusion of a second pump wave permits the excitation of up and down converted electromagnetic waves. Up converted electromagnetic waves can readily propagate in the surrounding regions of slightly higher plasma density without being absorbed.

We showed that within the decay limit, $k_0 > (2/3)(\mu\tau)^{1/2}$, all models treated here give similar results. We also showed that Akimoto (1988) and Abalde et al. (1998) models lead to same results concerning to the frequency of electromagnetic wave generation.

When the ion-acoustic wave is off resonance we verify that the inclusion of Langmuir wave daughters gives a broader region of instability compared to the Chian & Alves (1988) or the Glanz et al. (1993) model that includes only excited electromagnetic waves. The presence of a second pump wave with different amplitude from the first one, $r \neq 1$, a more realistic situation, brings a region of convective instability, not present for $r = 1$. The critical value of $k_T\lambda_D$ where the transition from purely growing to convective instability occurs decreases as r decreases. There is an evidence that there is a transition from hybrid modes and purely electrostatic or electromagnetic ones. In order to verify that we have to obtain the relative amplitude among the several daughter waves, which will be done in the near future.

There are a number of observational reports of a close correlation of Langmuir and ion- acoustic waves in connection with type III radio bursts (Lin et al. 1986; Gurnett et al. (1993). Recently, Thejappa & MacDowall (1998) presented experimental results for type III radio burst with clear evidence of occurrence of ion-acoustic waves in association with Langmuir waves. The theory presented in this paper provides a detailed picture of nonlinear wave-wave interaction processes which may be responsible for the excitation of type III solar radio emissions.

Acknowledgements. We thank Dr. M. Karlicky for valuable comments. This work is partially supported by FAPESP- Fundação de Amparo à Pesquisa do Estado de São Paulo, Brazil, and by CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico. A.C.-L.C. and F.B.R. wish to thank the CSSM, Adelaide University, for the warm hospitality.

References

Abalde, J. R., Alves, M. V., & Chian, A. C.-L. 1998, A&A, 331, L21
Akimoto, K. 1988, Phys. Fluids, 31, 538
Bárta, M., & Karlicky, M. 2000, A&A, 353, 757
Chian, A. C.-L., & Abalde, J. R. 1997, J. Plasma Phys., 57, 753
Chian, A. C.-L., & Alves, M. V. 1988, ApJ, 330, L77
Ginzburg, V. L., & Zheleznyakov, V. V. 1958, Sov. Astron., 2, 653
Glanz, J., Goldman, M., & Newman, D. L. 1993, Phys. Fluids, 5, 1101
Gurnett, D. A., Hospodarsky, G. B., Kurth, W. S., Williams, D. J., & Bolton, S. J. 1993, J. Geophys. Res., 98, 5631
Lashmore-Davies, C. N. 1974, Phys. Rev. Lett., 32, 289
Lin, R. P., Levedahal, W. K., Lotko, W., Gurnett, D. A., & Scarf, F. L. 1986, ApJ, 308, 854
Rizzato, F. B., & Chian, A. C.-L. 1992, J. Plasma Phys., 48, 71
Robinson, P. A. 1997, Rev. Mod. Phys., 69, 507
Thejappa, G., & MacDowall R. J. 1998, ApJ, 498, 465