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**ON THE IMPROVEMENT OF 1-LOOK SAR IMAGE SEGMENTATIONS WITH
MATHEMATICAL MORPHOLOGY**

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ABSTRACT

Synthetic Aperture Radar (SAR) images are an important source of information. This kind of imaging is little affected by adverse atmospheric conditions, such as rain, clouds, fog, etc., since it operates at frequencies other than the visible. Also, since the sensor is active and carries its own source of illumination, it can operate by night. The problem that arises with the use of this technology is a signal-dependent noise, called *speckle*. This kind of noise is common to all imaging devices that use coherent illumination, such as laser, microwaves, etc. One of the most useful techniques for image analysis is the segmentation. Using statistical modelling, two multiclass segmentation techniques for 1-look and linear detection SAR images are derived: the maximum likelihood and the Iterated Conditional Modes (ICM), both assuming multiplicative Rayleigh models for the data. Although the ICM segmentation yields significantly better results than the maximum likelihood segmentation, the 1-look linear detection case is noisy enough to deserve some improvement. Mathematical Morphology, a non linear approach to signal processing, is then used as a refinement technique in order to extract information.

1. INTRODUCTION

Image processing techniques have suffered an explosive growth in the last years. This is due to, among other reasons, the fact that more and more information from different sources, in the form of images, is available. It is known that about 50% of the images sent by environmental satellites has never been seen by human eyes... and that most of them carry precious information.

That information, when retrieved within an acceptable delay after its acquisition, is present in our daily life: from relevant information about the weather, to statistics about continental growth of crops that take part in decisions about international financial aids, including information for ships about drifting icebergs.

Therefore, it is becoming of crucial importance the ability to process images rapidly, and in such a manner that the relevant information within these images is accurately retrieved. Within this context, statistical ideas are becoming very important. Other tools, such as the ones derived from Artificial Intelligence, Mathematical Morphology, etc., are also contributing to the development of techniques for the processing and the interpretation of images.

SAR images are receiving an increasing attention^{20,24} mainly due to the several satellites already launched, not to mention the airborne missions. This kind of imaging, besides providing dielectrical and textural information of the targets, is little affected by adverse atmospheric conditions, such as fog, rain, clouds, etc. It also allows night imaging, since the sensor is active and carries its own illumination source.

The main problem with the use of SAR images is a kind of signal-dependent noise: *speckle* noise. It makes difficult the visual and the automatic interpretation of the data, since it produces a salt-and-pepper effect.

2. DEFINITIONS AND NOTATION

The indicator function of the set A is defined, and denoted, as:

$$1_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{else.} \end{cases}$$

The set of the real numbers is denoted $\mathbb{R}$, the set of positive real numbers $\mathbb{R}_+$, the set of natural numbers $\mathbb{N}$, and the set of the integers $\mathbb{Z}$. The cardinality of any denumerable set A shall be denoted $\#A$; the difference between the sets A and B shall be written $A - B = A \cap B^c$, where B^c is the complement of set B .

Random variables will be denoted with uppercase letters, and their outcomes with lowercase letters. A common underlying probability space will be assumed throughout this work: $(\Omega, \mathcal{A}, \text{Pr})$, where Ω denotes the sample set, $\mathcal{A}$ its σ -algebra, and Pr a probability. Therefore, random variables are measurable functions of the form $X : \Omega \rightarrow \mathcal{A}$, where $\mathcal{A}$ is the range of the function X .

3. STOCHASTIC MODELS FOR BAYESIAN IMAGE RESTORATION

Any observable image is supposed to be the outcome of a certain vector of random variables, which is indexed by the support of the image. This support is assumed to be a finite rectangle of the set Z^2 , denoted $S = \{0, \dots, m\} \times \{0, \dots, n\}$.

It will be assumed that every observable value is chosen from a finite or, at most, denumerable set. Another convenient, though not stringent, hypothesis is that this set is the same for every coordinate. Therefore, a *pixel* is defined as the pair (s, y_s) , with $s \in S$ and $y_s \in \Gamma'$. In this manner, the set of all the possible images is defined as $\Gamma = (\Gamma')^S$. The results presented in this work were obtained using $\Gamma' = \{0, \dots, 255\}$.

Assume that $X(\omega) = x$ is the outcome of the *ground truth*. This outcome is one element of the set $\Lambda = (\Lambda_{(s)})^S$. The *observed image*, denoted $Y(\omega) = y$ is a (possibly stochastic) transformation $\tau(x) = y \in \Gamma$. The aim of the Bayesian framework is assuming some (convenient) distributional properties for X and for τ and, therefore, estimating x throughout the observation of y . A detailed study of the transformation $\tau : \Lambda \rightarrow \Gamma$ is the subject of⁶.

3.1 The Potts–Strauss Model

The Potts–Strauss model is a special case of the class of Markov random field models. For a comprehensive account of information about these models, and their application to image synthesis, processing and analysis, the interested reader is referred to the works^{5,7,11,14,22}.

The Potts–Strauss model can be conveniently used as the *a priori* distribution for several Bayesian image problems. It serves as a convenient model for the contextual information. It can be assumed that the sets $\Lambda_{(s)}$ are of the form $\Lambda_{(s)} = \{0, 1, \dots, K - 1\}$ for every $s \in S$; typically, if $\#\Lambda_{(s)} = 2, 3$, up to, say, 8, the problem is considered as a segmentation one.

This model, for finite supports, can be equivalently defined in two manners: by a joint distribution and by the set of all the conditional distributions. The former is given by the expression:

$$\Pr(X = x) = \frac{1}{Z_\beta} \exp \left(\beta \sum_{s, t \in S: \|s - t\| = 1} 1_{[x_t]}(x_s) \right) \forall x \in \Lambda; \quad (1)$$

the latter by the set of equations $\Pr(X_s = x_s \mid X_{S-s} = x_{S-s}) \propto \exp(\beta \#\{t \in \partial_s : x_t = x_s\})$, for every $s \in S$, where “ $\propto$ ” denotes proportionality, $\|s - t\|$ is the euclidean distance between the coordinates $s, t \in S$, and the set denoted $\partial_s = \{t \in S : \|s - t\| = 1\}$ is the neighbor set of the coordinate $s \in S$. The parameter $\beta \in \mathbb{R}$ is often referred as the *attraction* parameter if it is positive, *repulsion* parameter otherwise. The constant Z_β is known as the *partition function*; it is defined by a sum over all the possible configurations (over all the elements of the set Λ) as:

$$Z_\beta = \sum_{x \in \Lambda} \exp(-H_\beta(x)),$$

where the function given by:

$$H_\beta(x) = -\beta \sum_{s, t \in S: \|s - t\| = 1} 1_{[x_t]}(x_s)$$

is often called the *energy* of the configuration $x \in \Lambda$.

It is possible to define this model with other neighbor sets, with different parameters, etc. (see¹⁵).

3.2 Pseudolikelihood Estimators

One of the major problems that arise with the use of Markovian distributions as *a priori* distributions in image processing is the need to choose its relevant parameters in a trial-and-error basis. These parameters are needed to completely specify the reconstruction and/or segmentation algorithms.

Given $X(\omega) = x$, outcome of the Potts–Strauss model, it is often desirable to estimate the parameter β . In general, it is impossible to solve the maximum likelihood equation, due to the presence of the partition function Z_β .

An alternative proposal¹⁹ is known as *maximum pseudolikelihood* estimation. It consists of maximizing the product of the conditional distributions.

In¹¹ these estimators are obtained in a computational tractable form. They are calculated for three different cases, namely, $K = 2$ (corresponding to the Ising model), $K = 3$, and $K \geq 4$. The set $W = \{1, \dots, m - 1\} \times \{1, \dots, n - 1\}$ is a convenient subset of the support S and, whenever necessary, the neighborhood of the point $s \in S$ shall be denoted $\partial_s = \{t, u, v, w\}$. The aforementioned estimators are given by the solution of the following equations:

• Case $K = 2$:

$$2(C_4^1 - C_0^1 + C_0^0 + C_4^1 + C_3^1) - C_1^1 + C_1^0 - 2(C_0^0 + 2C_4^1 + C_4^0)\Psi(4\hat{\beta}) - (C_1^0 + 2C_2^1 + C_3^0)\Psi(2\hat{\beta}) = 0,$$

where $C_k^t = \#\{s \in W : x_s = t, \sum_{i \in \partial_s} = k\}$ and $\Psi(t) = \exp(t)/(1 + \exp(t))$.

•Case $K = 3$:

$$\sum_{s \in W} v_s(x_s) - 4V^4\Psi(4\hat{\beta}) - V^3 \frac{3\exp(3\hat{\beta}) + \exp(\hat{\beta})}{\exp(3\hat{\beta}) + \exp(\hat{\beta})} - 2V^2,$$

where $v_s(t) = \#\{u \in \partial_s : x_u = t\}$, and:

$$V^4 = \#\{x_{\partial_s} \subset W : x_t = x_u = x_v = x_w\}, \quad V^3 = \#\{x_{\partial_s} \subset W : x_t = x_u = x_v \neq x_w\}, \quad V^2 = \#\{x_{\partial_s} \subset W : x_t = x_u \neq x_v = x_w\}$$

•Case $K \geq 4$:

$$\begin{aligned} \sum_{s \in W} v_s(x_s) - 4C_4 \frac{\exp(4\hat{\beta})}{\exp(4\hat{\beta}) + K} - C_3 \frac{3\exp(2\hat{\beta}) + \exp(\hat{\beta})}{\exp(3\hat{\beta}) + \exp(\hat{\beta}) + K - 1} - C_2 \frac{4\exp(2\hat{\beta})}{2\exp(2\hat{\beta}) + K - 1} + \\ - C_1 \frac{2(\exp(2\hat{\beta}) + \exp(\hat{\beta}))}{\exp(2\hat{\beta}) + \exp(\hat{\beta}) + K - 2} - C_0 \frac{4\exp(\hat{\beta})}{4\exp(\hat{\beta}) + K - 3} = 0, \end{aligned}$$

where $v_s(t) = \#\{u \in \partial_s : x_u = t\}$, and the constants C_i , for every $0 \leq i \leq 4$, are defined by:

$$\begin{aligned} C_4 &= \#\{x_{\partial_s} \subset W : x_t = x_u = x_v = x_w\}; \quad C_3 = \#\{x_{\partial_s} \subset W : x_t = x_u = x_v \neq x_w\}; \\ C_2 &= \#\{x_{\partial_s} \subset W : x_t = x_u \neq x_v \neq x_w \neq x_i\}; \quad C_1 = \#\{x_{\partial_s} \subset W : x_t = x_u \neq x_v \neq x_w\}; \\ C_0 &= \#\{x_{\partial_s} \subset W : x_i \neq x_j, \forall i \neq j\}. \end{aligned}$$

It is important to notice that all the involved constants C_k^t , V^k and C_i can be determined with only one sequential scanning of the sample $X(\omega) = x$ for every case of K ; therefore, the determination of $\hat{\beta}$ can be performed in a computationally quite inexpensive manner.

4. ELEMENTS OF MATHEMATICAL MORPHOLOGY

The definitions developed here, which are the transformations that are required in this paper, were extracted from¹. The details of the toolbox are found in³. An introduction of Mathematical Morphology can be found in^{2,16,17,18,23}.

Let Z be the set of integers. Let E be the a rectangle of Z^2 and K be the interval $[0, k - 1]$ of Z , with $k > 0$. The collection of functions from E to K will represent the gray-scale image of interest (the binary images are seen like a particular case of grey-scale images). Such collection will be denoted K^E , and f, g, f_1 and f_2 will denote generic elements of K^E . The intersection of f_1 e f_2 , denoted $f_1 \wedge f_2$, is the function in K^E given, for any $x \in E$, by $(f_1 \wedge f_2)(x) = \min\{f_1(x), f_2(x)\}$; the union of f_1 and f_2 , denoted $f_1 \vee f_2$, is the function in K^E given, for any $x \in E$, by $(f_1 \vee f_2)(x) = \max\{f_1(x), f_2(x)\}$.

The definitions of two important subclasses of operations, namely dilations and erosions, are recalled using the fact that $(Z^2, +)$ forms an Abelian group.

Let B be a subset of Z^2 , called structural element. The translation of B by any vector h in Z^2 , is denoted B_h ; that is, $B_h = \{x + h : x \in B\}$. The transpose of B is denoted B' , that is, $B' = \{-x : x \in B\}$. The complement of B is denoted B^c , that is, $B^c = \{x : x \notin B\}$. The dilation of f by B is the function $\delta_B(f)$ in K^E given, for any $x \in E$, by $\delta_B(f)(x) = \max\{f(y) : y \in B'_x \cap E\}$. The erosion of f by B is the function $\varepsilon_B(f)$ in K^E , given, for any $x \in E$, by $\varepsilon_B(f)(x) = \min\{f(y) : y \in B_x \cap E\}$. The two transformations δ_B and ε_B are called, respectively, dilation and erosion by B . The Minkowski addition of the two subsets A and B of Z^2 is a subset $A \oplus B$ of Z^2 , given by $A \oplus B = \bigcup \{A_b : b \in B\}$.

Let n be an integer larger then zero; denote nB the subset of Z^2 given by the $n - 1$ successive compositions $nB = (... (B \oplus B) ... \oplus B) \oplus B$.

The transformations γ_B and ϕ_B from K^E to K^E , given by the following compositions $\gamma_B = \delta_B \varepsilon_B$ and $\phi_B = \varepsilon_B \delta_B$ are called, respectively, the (morphological) opening and closing by B .

5. SAR IMAGE FORMATION AND THE ICM SEGMENTATION TECHNIQUE

A common model for homogeneous regions within SAR images is the multiplicative¹⁰. For a simplified version of this model, it can be assumed that the observations, given the classes, obey a Rayleigh distribution and that are independent from coordinate to coordinate.

The random variable Y is said to be Rayleigh distributed with parameter $\lambda \in \mathbb{R}_+$ if its cumulative distribution function is given by $F_Y(x; \lambda) = (1 - \exp(-x^2/(2\lambda^2)))1_{\mathbb{R}_+}(x)$. Therefore, its density is given by $f_Y(x; \lambda) = x/\lambda^2 \exp(-x^2/(2\lambda^2))1_{\mathbb{R}_+}(x)$. This case is denoted $Y \sim \mathcal{R}(\lambda)$.

5.1 Multiplicative Model for SAR Image Formation

Let $X(\omega) = x = [x_s]_{s \in S}$ be the outcome of the *a priori* distribution that models the ground truth. Assume that X has the Potts–Strauss distribution with $\Lambda_{(s)} = \{\lambda_1, \dots, \lambda_K\}$ for every $s \in S$ and $0 < \lambda_1 < \dots < \lambda_K < \infty$. The adopted model for image formation is given by:

$$\Pr(y; \lambda_{x_{s_1}}, \dots, \lambda_{x_{s_S}}) = \prod_{s \in S} \frac{y_s}{\lambda_{x_s}^2} \exp\left(-\frac{y_s^2}{2\lambda_{x_s}^2}\right) 1_{\mathbb{R}_+}(y_s), \quad (2)$$

i.e., it is assumed that the observations, given the classes, are the outcomes of independent Rayleigh distributed random variables, and that the parameters of these random variables depend on the class (in fact, what is observed is a truncated and discretized version of every observation y_s).

Another distribution that appears in the modelling of non-homogeneous areas^{25,26} is the amplitude K distribution that, for the considered data, is characterized by the following density:

$$f_{Z_A}(x; \alpha, \beta, 1) = \frac{4\alpha\beta}{\Gamma(\alpha)\beta} \left(\frac{\alpha x^2}{\beta}\right)^{(\alpha-1)/2} K_{\alpha-1}\left(2x\sqrt{\frac{\alpha}{\beta}}\right) 1_{\mathbb{R}_+}(x),$$

where K_ν is the modified Bessel function of the third kind and order ν . Whenever optical data are considered, a common hypothesis for their distribution is the assumption of normality. These three distributions are fitted to the data through the use of goodness-of-fit tests, and the results are presented in Table 1.

It is interesting to notice that this modelling also allows the design of specialized filters for speckle noise reduction^{13,21}.

5.2 The Bayesian Restoration of Images

Given the observation of $Y(\omega) = y = [y_s]_{s \in S}$, it is desirable to estimate the unobserved outcome $X(\omega) = x = [x_s]_{s \in S}$ of the *a priori* distribution. Several techniques, that use the presented modelling, have been proposed in the literature; all of them are estimators of x based on the observation of y (see, for example, the references^{4,5,7,14,22}).

Two estimators have received great attention, namely, the *Maximum a Posteriori* (MAP) and the *Marginal Posterior Modes* (MPM). They are defined as:

$$\hat{x}_{\text{MAP}} = \max_{x \in \Lambda}^{-1} \Pr(x|y), \quad \text{and} \\ \hat{x}_{\text{MPM}} = \left[\max_{x_s \in \Lambda_{(s)}}^{-1} \Pr(x_s|y) \right]_{s \in S}.$$

The problem with the use of these estimators appear due to the fact that they cannot be calculated directly. They have to be approximated by iterative stochastic techniques, whenever the *a priori* distribution is non-trivial. In order to approximate the former, *simulated annealing* has to be used. The latter estimator can be approximated simulating events from the distribution given by $\Pr(X|y)$, and recording in every coordinate the most frequently observed value. An interesting connection between those Bayesian techniques that use a non-trivial *a priori* distribution, and Mathematical Morphology can be found in²⁷.

5.3 The ICM Algorithm

This algorithm, originally proposed by Besag⁴ as a manner of approximating the MAP estimator of x given y , with Markovian *a priori* distribution, soon gained a position of estimator as itself^{5,7,8,9,11,12}.

This iterative technique consists of the maximization of the conditional distributions, for every coordinate, until there is evidence of convergence. It can be proved that, if implemented in a non-parallel manner, it produces a sequence of solutions with non-decreasing likelihood, i.e. if $\hat{x}(k)$ denotes the estimator (reconstructed or segmented image) in the k -th iteration, then it holds that:

$$\Pr(\hat{x}(0)|y) \leq \Pr(\hat{x}(1)|y) \leq \Pr(\hat{x}(2)|y) \leq \dots$$

The k -th iteration consists of replacing every $[\hat{x}(k)]_{s \in S}$ by $\max_{x_s \in \Lambda_{(s)}}^{-1} \Pr(x_s | y, x_{S \ominus \{s\}})$. Using the properties of the distribution expressed by equation (1), it is possible to simplify the dependence on the reconstructed image, yielding $\max_{x_s \in \Lambda_{(s)}}^{-1} \Pr(x_s | y, x_{\partial_s})$. Now, using the properties of the multiplicative model presented in Section 5.1, this last expression simplifies further, yielding $\max_{x_s \in \Lambda_{(s)}}^{-1} \Pr(x_s | y_s, x_{\partial_s})$ and, simplifying still further, $\max_{x_s \in \Lambda_{(s)}}^{-1} (\Pr(x_s | y_s) \cdot \Pr(x_s | x_{\partial_s}))$. In this manner, the ICM algorithm can be expressed as: the k -th iteration consists of replacing every $[\hat{x}(k)]_{s \in S}$ by $\max_{x_s \in \Lambda_{(s)}}^{-1} \Pr(x_s | y_s, x_{\partial_s})$, where:

$$\Pr(x_s | y_s, x_{\partial_s}) \propto \exp(\beta \#\{t \in \partial_s : x_t = x_s\}) \cdot \frac{y_s}{\lambda_{x_s}^2} \exp\left(-\frac{y_s^2}{2\lambda_{x_s}^2}\right).$$

In order to avoid the effects of sequential scanning, pointed out in⁷, the implementation here presented makes sequential scanings in subsets that form a partition of the support. To illustrate this idea, the order of visits to a toy support of the form $S = \{1, \dots, 6\} \times \{1, \dots, 6\}$ is shown in Figure 1.

1	10	2	11	3	12
19	28	20	29	21	30
4	13	5	14	6	15
22	31	23	32	24	33
7	16	8	17	9	18
25	34	26	35	27	36

Fig. 1. Sequence of updatings of the ICM algorithm to a squared support of size 36.

The presented version of this algorithm is automatic, in the sense that it does not require the use of unknown parameters. Every iteration uses as the parameter of the *a priori* distribution the result of estimating β with the pseudolikelihood estimator $\hat{\beta}$, based on the previous solution, i.e., $\hat{\beta}(k) = \hat{\beta}(\hat{x}(k-1))$.

The first configuration, $\hat{x}(0)$, is, by definition, any possible configuration. In this paper, these configurations were obtained by the use of the maximum likelihood classification technique under the Rayleigh distribution¹¹.

The maximum likelihood classification criterion can be expressed in the following manner: assume that the observation $y \in \mathbb{R}_+$ is to be attributed to one of the following two populations, P_1 characterized by a $\mathcal{R}(\lambda_1)$ distribution, or P_2 characterized by a $\mathcal{R}(\lambda_2)$ distribution, with $\lambda_1 < \lambda_2$. The point $x^* \in \mathbb{R}_+$ where $f_Y(x^*; \lambda_1) = f_Y(x^*; \lambda_2)$ is given by:

$$x^* = \sqrt{\frac{4 \ln\left(\frac{\lambda_2}{\lambda_1}\right)}{\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}}}.$$

Therefore, this criterion is given by the rule: assign y to P_1 if $y < x^*$; else assign y to P_2 . The case $y = x^*$ could be ignored since, at least theoretically, the probability of observing such value is identically zero for any random variable that admits a (strictly continuous) density. The generalization of this rule for more than two classes is immediate.

Figure 2 shows the result of fitting two Rayleigh densities (continuous and dashed lines) over the histograms (“+” and “o”, resp.) of homogeneous regions, obtained by visual inspection of Figure 3. The parameters λ_1 and λ_2 were obtained applying the maximum likelihood estimators to those samples, given by:

$$\hat{\lambda} = \sqrt{\frac{1}{2n} \sum_{j=1}^n y_j^2},$$

yielding $\hat{\lambda}_1 = 28.9$ and $\hat{\lambda}_2 = 77.9$; therefore, $x^* \approx 62$ is the value that discriminates class P_1 (dark area) from class P_2 (bright area).

Since the validity of the presented segmentation technique heavily depends on the assumption of Rayleigh distributions for the observations, two goodness-of-fit tests were applied to the aforementioned samples: the Kolmogorov-Smirnov (KS) and the χ^2 (C2) tests. The p -values of the hypothesis of Rayleigh, amplitude K and Normal distributions is shown in Table 1, show-

ing no evidence of lack of adequacy of the Rayleigh distribution for the considered image, at least at the 1% level of significance. The same table clearly shows that the Normal distribution is not a good assumption for these data.

The convergence criteria used to obtain the results presented in Section 7 was the strongest: stop whenever there is no change from one configuration to the next.

TAB. 1. PERCENTUAL P-VALUES OF TWO GOODNESS-OF-FIT TESTS OVER HOMOGENEOUS REGIONS

DIST.	Rayleigh		Amplitude <i>K</i>		Normal	
AREA	KS	C2	KS	C2	KS	C2
Dark	2.28	26.53	2.28	26.53	0.00	0.00
Bright	80.93	48.12	0.00	0.00	0.00	0.00

Fig. 2. Histograms and fitted densities over samples of homogeneous regions.

6. APPLICATION OF MATHEMATICAL MORPHOLOGY TOOLS TO SEGMENTATION IMPROVEMENT

Although the ICM segmentation yields significantly better results than the maximum likelihood segmentation, as can be shown comparing Figures 5 and 6 (for binary segmentations) and Figures 9 and 10 (for three-class segmentations), the 1-look linear detection case is noisy enough to deserve some improvement. With the use of tools derived from the Mathematical Morphology framework, better results are obtained than those solely reached by the use of statistical segmentation.

Mathematical Morphology operations are used in this work, as a refinement technique aiming at the uniformization of the statistical segmentations. The results of applying the aforementioned segmentation algorithms are, then, improved with the use of morphological operations. It is shown that, starting with a statistical segmentation and then applying operations derived from the Mathematical Morphology framework, it is possible to generate hybrid (statistical/morphological) products that are more adequate for image understanding than the original ones. It is also shown that the use of the ICM algorithm yields an initial segmentation that is more easily improved. Observing the resulting ICM and maximum likelihood segmented images, it can be seen that both of them have lots of small regions that are not correctly attributed. The tools of Mathematical Morphology will be used to uniformize the classes.

Observing the segmentations obtained by the maximum likelihood and the ICM algorithms, both for two and three classes segmentations (Figures 5, 6, 9 and 10), it can be seen that there are some small spurious classes within large area classes. The opening transformation could remove small regions with high grey scale levels, whilst the closing transformation could remove small regions with low grey levels. Then, when these two operations are combined, those two characteristics could be removed; since the spurious segmentations have this characteristics, these transformations could be adequate for the improvement of the segmentations. The combination of openings and closings is called "alternating sequential filtering"; the results of applying

this technique to two and three classes and maximum likelihood and ICM segmentations is shown in Figures 7, 8, 11 and 12, respectively.

Let f be the image segmented by the ICM or maximum likelihood technique in any number of classes. Let f_1 be the image resulting from applying the alternating sequential filtering technique obtained by $f_1 = \eta_{B_6}(\eta_{B_5}(\eta_{B_4}(\eta_{B_3}(\eta_{B_2}(\eta_{B_1}(f))))))$, where η_B is the result of composing the operations of opening and closing by the structuring element B , i.e. $\eta_B = \gamma_B \circ \phi_B$, and the sets B_1, B_2, B_3, B_4, B_5 and B_6 are the structuring elements, defined by:

$$B_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

and $B_3 = 2B_1; B_4 = 2B_2; B_5 = 3B_1; B_6 = 3B_2$. This filter is presented in³.

7. RESULTS

The original image (Figure 3) is a SAR580 image over the region of Freiburg, Germany (512×512 pixels). This is an airborne sensor, and the image shown was obtained in the L-band, HH polarization. A visual interpretation of this image shows two main classes: a bright one, corresponding to forest, and a dark one, corresponding to bare soil. Some very bright spots in the lower right corner of the image are due to the typical high return of buildings, an effect known as "double bounce backscatter". Figure 4 is an image of the same region, though obtained in another frequency, namely, band X; it is shown, and it was used, after histogram equalization, and it served as an additional source of visual information for the discrimination of homogeneous regions. For the use of both bands, after suitable transformations, in multiclass and multiband SAR image segmentation, the reader is referred to^{8,11}.

Fig. 3. Original image: L-band, HH polarization.

Fig. 4. Additional image: X-band, HH polarization.

The following figures show the results of applying the maximum likelihood and ICM segmentation algorithms to the image shown in Figure 3. Comparing Figures 5 and 6 it is evident that though the ICM segmentation retains more contextual information, it would be desirable to improve it. Figures 7 and 8 show the results of applying the alternating sequential filter presented above, and it could be said that the use of this morphological operation yields better results than the original segmentations, since the borders between regions are more clearly defined.

The ICM segmentation (Figure 6) is visually better than the maximum likelihood one (Figure 5), since the "grain size" is larger in the former, but in both cases the spurious segmentations make hard the detection of boundaries between classes. With

the use of the alternating sequential filter defined in Section 6, both segmentations are improved. It is interesting, though, to notice that the ICM-improved segmentation retains better some spatial features, as can be seen comparing the area close to the middle portion of the right border in both images.

Though it is not quite clear how to extract three classes from the original image, the presented technique was also tested with maximum likelihood and ICM three classes segmentations; the information required to obtain these segmentations was obtained from the L (Figure 3) and X bands (Figure 4). This last image is shown equalized for purposes of visualization. It is interesting to note that though there is not much visual evidence of three classes, the ICM-improved segmentation (Figure 12) allows a discrimination of edges between regions which is not quite evident in the other segmentations.

In order to make a quantitative, rather than the aforementioned qualitative, comparison between the four presented segmentations (maximum likelihood, ICM and its morphological improvements) a ground-truth map could be used.

8. CONCLUSIONS

In this paper it was shown that the use of an alternating sequential filter, derived from the Mathematical Morphology framework, can be successfully used to improve one look amplitude SAR segmented images. Also, the adequacy of a Markovian segmentation technique, the ICM multiclass algorithm, is shown since the improved versions of its segmentations are better than the improved versions of the segmentations obtained with the maximum likelihood algorithm. The presented version of the ICM algorithm does not require trial-and-error choice of parameters, since every required quantity is estimated from the data; in particular, the parameters of the Potts-Strauss model are obtained iteratively using the maximum pseudolikelihood estimators. Both maximum likelihood and ICM techniques are adapted to the multiplicative model and linear detection, and the hypothesis of Rayleigh distribution is tested with the Kolmogorov-Smirnov and χ^2 goodness-of-fit tests over homogeneous areas.

9. COMPUTATIONAL INFORMATION

The results of applying the Mathematical Morphology filters were obtained using the Mathematical Morphology Toolbox developed for the KHOROS system and presented in³. The software for maximum likelihood and ICM segmentation of one look amplitude SAR images is available, upon request, to E-Mail frery@dpi.inpe.br.

Fig. 5. Binary maximum likelihood segmentation.

Fig. 6. Binary ICM segmentation.

Fig. 7. Morphological improvement of Figure 5.

Fig. 8. Morphological improvement of Figure 6.

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Fig. 9. Maximum likelihood segmentation in three classes.

Fig. 10. ICM segmentation in three classes.

Fig. 11. Morphological improvement of Figure 9.

Fig. 12. Morphological improvement of Figure 10.

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