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**IMPROVING THE RATE MONOTONIC AND THE FIRST
DEADLINE FIRST SCHEDULERS FOR REAL TIME
SIMULATION AND CONTROL OF AEROSPACE VEHICLES**

Gilberto da Cunha Trivelato*
Marcelo Lopes de Oliveira e Souza

*EMBRAER- Empresa Brasileira de Aeronáutica

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Gilberto da Cunha Trivelato, M.Sc.

EMBRAER – Empresa Brasileira de Aeronáutica S. A.
P.O. Box 8050
12227-901 - S. José dos Campos – SP – Brasil
trivelato@embraer.com.br

Marcelo Lopes de Oliveira e Souza, Ph.D.

INPE – National Institute for Space Research
DMC – Division of Space Mechanics and Control
Av. dos Astronautas, 1758
12227-010 - S. José dos Campos – SP – Brasil
marcelo@dem.inpe.br

ABSTRACT: *In this work we discuss and improve the rate monotonic scheduler and the first deadline first scheduler for real time simulation and control. These algorithms are largely known and discussed in the real time literature but in such literature: 1) some aspects of their original propositions are/remain incomplete or hidden; 2) there are even some counter-examples and critiques to them made in recent works. Correspondingly, in this work: 1) we discuss and clarify some aspects of their original propositions and we improve such propositions by explicitly writing down some of their incomplete or even hidden passages and transformations; 2) we also discuss and clarify some mistakes done in those counter-examples and critiques, by replacing them with new examples that confirm the algorithms and illustrate the points discussed, clarified, and improved. We also suggest some possible extensions of those algorithms and their uses in real time simulation and control of aerospace vehicles, especially aircrafts and satellites.*

KEYWORDS: *Scheduling Techniques, Rate Monotonic Algorithm, First Deadline First Algorithm, Real Time Simulation, Real Time Control, Aerospace Vehicles*

INTRODUCTION

Even today with incredible and powerful computers, experience shows that we are always looking for more processing time for new and more complex tasks. There are some special embedded applications where we cannot replace and/or upgrade computers anytime but the environment always changes: an operation flight program for a fighter in a complex and dynamic defense system, a spaceship or spacecraft in an interplanetary trip flying through debris, or a future microcomputer inside human body as a complementary part of its defense or behavior systems. These systems need to be “hard real time”.

“Hard real time” system is a system where the service must be guaranteed for each task before the next request for the same task. A short and good bibliography revision about real time systems is done by Farines et al.¹. Then the scheduling program (“scheduler”) in a single processor is always studied

from the viewpoint of its most important constraint: guaranteed service. In general we consider a parallel processor machine as a “single” processor machine with finite processing resources. Fujimoto² presents a detailed study about parallel and distributed simulation. A Schedulers are the rules that determine the time for the set of tasks to be executed. There are many schedulers in the literature including the Rate Monotonic Scheduler (RMS) and the First Deadline First Scheduler (FDFS) or Earliest Deadline First Scheduler (EDS).

In this work we discuss and improve the rate monotonic scheduler and the first deadline first scheduler for real time simulation and control. These algorithms are largely known and discussed in the real time literature but in such literature: 1) some aspects of their original propositions are/remain incomplete or hidden; 2) there are even some counter-examples and critiques to them made in recent works. Correspondingly, in this work:

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1) we discuss and clarify some aspects of their original propositions and we improve such propositions by explicitly writing down some of their incomplete or even hidden passages and transformations; 2) we also discuss and clarify some mistakes done in those counter-examples and critiques, by replacing them with new examples that confirm the algorithms and illustrate the points discussed, clarified, and improved. We also suggest some possible extensions of those algorithms and their uses in real time simulation and control of aerospace vehicles, especially aircrafts and satellites.

ENVIRONMENT

The hard-real time environment has the same **assumptions** presented in Liu and Layland³ as follows: “(A1) The requests for all tasks for which hard deadlines exist are periodic, with constant intervals between requests.

(A2) Deadlines consist of run-ability constraints only – i.e. each task must be completed before the next request for it occurs.

(A3) The tasks are independent in that requests for a certain task do not depend on the initiation or the completion of requests for other tasks.

(A4) Run-time for each task is constant for that task and does not vary with time. Run-time here refers to the time that is taken by a processor to execute the task without interruption.

(A5) Any non-periodic tasks in the system are special; they are initialization or failure-recovery routines; they displace periodic tasks while they themselves are being run, and do not themselves have hard, critical deadlines.”

As a consequence, we can have a complete characterization of a task by its request period (T_i) and their run time (C_i). Then, in the same way as defined by Liu and Layland³, “in this paper we shall use $t_1, t_2, \dots, t_m$ to denote m periodic tasks with their request periods being $T_1, T_2, \dots, T_m$ and their run-times $C_1, C_2, \dots, C_m$, respectively. The request rate of a task is defined to be the reciprocal of its request period.”

A FIXED PRIORITY SCHEDULING ALGORITHM

The basic definitions are the same presented in Liu and Layland³. A **static scheduler** assigns a priority to each task in the design time and it remains the same. But a **dynamic scheduler** assigns such priority in run-time,

so it is variable. The **Rate Monotonic Scheduler** (RMS) is static and it assigns a fixed priority to each process based on its period: the shorter the period the higher the priority. But a **First Deadline First Scheduler** (FDFS) or **Earliest Deadline First** (EDF) is a dynamic scheduler that has explicit information about deadlines and assigns higher priority to the process with the nearest deadline. A **preemptive scheduler** immediately suspends the current task and starts a higher priority task whenever it is requested. “The **deadline** of a request for a task is defined to be the time of the next request for the same task.” In a scheduling algorithm, “we say that an **overflow** occurs at time t if t is the deadline of an unfulfilled request”. “A **critical instant** for a task is defined to be an instant at which a request for that task will have the largest response time”. The next critical instant will happen in the minimum common multiple (MCM) for all tasks. “A **critical time zone** for a task is the time interval between a critical instant and the end of the response to the corresponding request of the task”. A set of tasks “**fully utilize** the processor if the priority assignment is feasible for the set and if an increase in the run-time of any of the tasks in the set will make the priority assignment infeasible”. The task **skew** is the time delay between the beginning of T_i and C_i .

Liu and Layland³ proposed **theorem 1**: *A critical instant for any task occurs whenever the task is requested simultaneously with requests for all higher priority tasks.*

Proof: Liu and Layland³. The proof is clear and easy to understand. The result in theorem 1 also suggests a priority assignment that is optimum in the sense that will be stated in theorem 2. “Such an assignment of priorities will be known as the rate monotonic priority assignment. As it turns out, such a priority assignment is optimum in the sense that no other fixed priority assignment rule can schedule a task set that cannot be scheduled by the rate-monotonic priority assignment.” “If the requests for all tasks at their critical instants are fulfilled before their respective deadlines, then the scheduling algorithm is feasible.”

Liu and Layland³ proposed **theorem 2**: *If a feasible priority assignment exists for some task set, the rate monotonic priority assignment is feasible for that task.*

Proof: See Liu and Layland³. The proof is clear and easy to understand, but we clarify that it is only applied to the fixed schedulers family.

PROCESSOR UTILIZATION FACTOR

Liu and Layland³ defined processor utilization factor to be the fraction of processor time spent in the execution of the task set. Since (C_i/T_i) is the fraction of processor time spent in executing task t_i , for m tasks, the utilization factor is:

$$U(C_i, T_i) = \sum_{i=1}^m (C_i/T_i). \quad (1)$$

However, in this definition, we should clarify, that (1) can be rewritten as follows:

$$U(C_i, T_i, M) = \sum_{i=1}^m \{(M/T_i) \cdot (C_i/M)\} \quad (2)$$

where M is the minimum common multiple (MCM) among all task periods. If they are mutually prime, then $M = T_1 T_2 \dots T_m$. Defining $n_i = M/T_i$ as a number of times that task i appears in the interval time M , we get:

$$U(n_i, C_i, M) = \{\sum_{i=1}^m (n_i \cdot C_i)\} / M. \quad (3)$$

We can say that the processor utilization factor is the percentage of time spent by processor executing all tasks in any interval time M . If there is no common factor between any (T_i, T_j) then, when m grows, M grows faster. Even if we could define a set of tasks with all skews equal to zero that fully utilize the processor considering only the interval T_m , the utilization factor has real meaning only in the time interval M .

Liu and Layland³ proposed **theorem 3**: For a set of two tasks with fixed priority assignments, the least upper bound (lub) to the processor utilization factor is $U_{\text{lub}} = 2 \cdot (2^{1/2} - 1)$.

We clarify the proof including the graphical representation of the task set in both cases.

Let t_1, t_2 be two tasks with periods being T_1 and T_2 and run-times being C_1 and C_2 , respectively. Assume that $T_1 > T_2$ w.o.l.o.g. According to the rate-monotonic priority assignment t_1 has higher priority than t_2 . In a critical time zone of t_2 , there are $\lceil T_2/T_1 \rceil$ ("ceiling" of $T_2/T_1 = \text{smallest integer} \geq T_2/T_1$) requests for t_1 . If we adjust C_2 to fully utilize the processor we have two cases:

Case 1: all requests of t_1 within the critical time zone of T_2 are completed before the second t_2 request – Figure 1.

Using the results from Liu and Layland³, where $C_2 = C_2(C_1, T_1, T_2)$, we have:

$$U(C_1, T_1, T_2) = 1 + C_1 \underbrace{\left[(1/T_1) - (1/T_2) \lceil T_2/T_1 \rceil \right]}_{\leq 0} \quad (4)$$

In this case, the processor utilization factor $U(C_1, T_1, T_2)$ is monotonically decreasing with C_1 .

Case 2: the execution of $\lceil T_2/T_1 \rceil^{\text{th}}$ request for t_1 overlaps the second t_2 request - Figure 2.

Using the results from Liu and Layland³, where $C_2 = C_2(C_1, T_1, T_2)$ and $\lfloor T_2/T_1 \rfloor$ is "floor" of $T_2/T_1 = \text{greatest integer} \leq T_2/T_1$, we have:

$$U = (T_1/T_2) \lfloor T_2/T_1 \rfloor + C_1 \underbrace{\left[(1/T_1) - (1/T_2) \lfloor T_2/T_1 \rfloor \right]}_{\geq 0}. \quad (5)$$

Figure 1 – Task scheduling for case 1, with $t_1 = \{T_1 = 6, C_1 = 3\}$, $t_2 = \{T_2 = 16, C_2 = 7\}$ time units.

Figure 2 – Task scheduling for case 2, with $\mathbf{t}_1 = \{T_1 = 6, C_1 = 3\}, \mathbf{t}_2 = \{T_2 = 14, C_2 = 6\}$ time units.

In this case, the processor utilization factor $U(C_1, T_1, T_2)$ is monotonically increasing with C_1 .

The minimum of $U(C_1, T_1, T_2)$ clearly occurs at the boundary between (3) and (4). Then:

$$C_1 = T_2 - T_1 \lfloor T_2 / T_1 \rfloor. \quad (6)$$

As shown in Liu and Layland³ this value is:

$$U(T_1, T_2) = 1 - (T_1/T_2) \lfloor T_2/T_1 \rfloor - (T_2/T_1) \lfloor T_2/T_1 \rfloor \quad (7)$$

and the least upper bound of $U(T_1, T_2)$ for all T_1, T_2 is numerically $U_{\text{lub}} = 2 \cdot (2^{1/2} - 1) \approx 0.83$.

Other Liu and Layland³ proposition is **theorem 4**: For a set of m tasks with fixed priority order, and the restriction that the ratio between any two request periods is less than 2, the least upper bound to the processor utilization factor is $U_{\text{lub}} = m \cdot (2^{1/m} - 1)$.

Proof: Let $\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_m$ to denote m periodic tasks.

Let $C_1, C_2, \dots, C_m$ be the run-times of the tasks that fully utilize the processor and minimize the processor utilization factor. Assume that $T_1 < T_2 < \dots < T_{m-1} < T_m$. Let $U(C_i, T_i)$ denote the processor utilization factor.

Lets proof the theorem as Liu and Layland³ and illustrate it with $\mathbf{t}_1 = \{T_1 = 8, C_1^{\text{lub}} = 3\}$, $\mathbf{t}_2 = \{T_2 = 11, C_2^{\text{lub}} = 4\}$ and $\mathbf{t}_3 = \{T_3 = 15, C_3^{\text{lub}} = 1\}$.

We wish to show that (lemma 1): $C_1^{\text{lub}} = T_2 - T_1$.

Suppose that $C_1 = T_2 - T_1 + \Delta$, $\Delta > 0$, $\Delta = 1$ then, we have a new task $\mathbf{t}_1 = \{T_1 = 8, C_1 = 4\}$, $\mathbf{t}_2 = \{T_2 = 11, C_2 = 3\}$ and $\mathbf{t}_3 = \{T_3 = 15, C_3 = 1\}$ as shown in Figure 3 and clearly fully utilize the processor. It is equivalent to transfer Δ from task $\mathbf{t}_2$

to task $\mathbf{t}_1$. The first Δ from C_2^{lub} compensates directly C_1 . The second Δ , added to second C_1 , postpone the second C_2 by Δ with no overflow because it is the same amount left from the end.

Lets rewrite the task as $C'_1, C'_2, \dots, C'_m$, where:

$$\begin{aligned} C'_1 &= T_2 - T_1 \\ C'_2 &= C_2 + \Delta \\ C'_3 &= C_3 \\ &\vdots \\ C'_{m-1} &= C_{m-1} \\ C'_m &= C_m \end{aligned}$$

If U' denotes the corresponding utilization factor, we have:

$$U - U' = (\Delta/T_1) - (\Delta/T_2) > 0. \quad (8)$$

Some important facts that are not in Liu and Layland³, but are introduced here, are that: if $\mathbf{t}_i$, $i = 2, \dots, m$, satisfies $C_i > \Delta$, we can transfer Δ from any task $\mathbf{t}_i$ ($1 < i \leq m$) to $\mathbf{t}_1$ and the result is still valid, i.e., $U_{\text{lub}} - U^i = (\Delta/T_1) - (\Delta/T_i) > 0$. And more, $U_{\text{lub}} - U^i < U_{\text{lub}} - U^{i+1}$ because $T_i < T_{i+1}$.

Alternatively, suppose that $C_1 = T_2 - T_1 - \Delta$, $\Delta > 0$, $\Delta = 1$ then, we have a new task $\mathbf{t}_1 = \{T_1 = 8, C_1 = 2\}$, $\mathbf{t}_2 = \{T_2 = 11, C_2 = 6\}$ and $\mathbf{t}_3 = \{T_3 = 15, C_3 = 1\}$ as shown in Figure 4 and clearly fully utilize the processor. It is equivalent to transfer Δ from task $\mathbf{t}_1$ to task $\mathbf{t}_2$. Because, under

Figure 3 – Full utilization with $C_1 = T_2 - T_1 + \Delta$.

Figure 4 - Full utilization with $C_1 = T_2 - T_1 - \Delta$.

theorem hypothesis, there are always two C_1^{lub} executions before the second t_2 request in time interval T_2 , we must add in C_2 twice the Δ value subtracted from C_1^{lub} . In Figure 4 we can verify that in the first time interval T_2 we have 2 times C_1 executions, in the first interval T_3 we also have 2 times C_2 executions, and first C_3 is always executed just before a completion of first time interval T_1 .

Lets rewrite the task as $C_1'', C_2'', \dots, C_m''$, where:

$$\begin{aligned} C_1'' &= T_2 - T_1 \\ C_2'' &= C_2 - 2\Delta \\ C_3'' &= C_3 \end{aligned}$$

$$\begin{aligned} &\vdots \\ C_{m-1}'' &= C_{m-1} \\ C_m'' &= C_m \end{aligned}$$

If U'' denotes the corresponding utilization factor, we have:

$$U - U'' = -(\Delta/T_1) + 2.(\Delta/T_2) > 0. \quad (9)$$

Some important facts that are not in Liu and Layland³, but are introduced here, are that we can cut Δ from task t_1 and add 2Δ to any t_i ($1 < i \leq m$) and the result is still valid, i.e., $U_{\text{lub}} - U^i = -(\Delta/T_1) + (2\Delta/T_i) > 0$, and again, $U_{\text{lub}} - U^i < U_{\text{lub}} - U^{i+1}$.

Therefore, if indeed U is the minimum of full utilization factors, then $C_1 = T_2 - T_1$. In the previous numerical example we can verify that two modified task sets $C'_1, C'_2, \dots, C'_m$ and $C''_1, C''_2, \dots, C''_m$ satisfy this condition and are equal to $C_1^{\text{lub}}, C_2^{\text{lub}}, \dots, C_m^{\text{lub}}$.

Now we wish to show that (lemma 2): $C_m^{\text{lub}} = T_m - T_{m-1}$.

Liu and Layland³ did say, “in a similar way we can prove that”, as shown in Liu and Layland³:

$$C_2 = T_3 - T_2$$

$$C_3 = T_4 - T_3$$

$$\vdots$$

$$C_{m-1} = T_m - T_{m-1}$$

$$C_m = T_m - 2 \cdot (C_1 + C_2 + \dots + C_{m-1})$$

But they did not prove it in Liu and Layland³. So, let's clarify this using Figure 5.

The rest of the proof is as in Liu and Layland³; and it gives as the important result:

$$U_{\text{lub}(m)}^m = m \cdot (2^{1/m} - 1). \quad (10)$$

Devillers and Goossens⁴ used a numerical example $\mathbf{t}_1 = \{T_1 = 8, C_1 = 2\}, \mathbf{t}_2 = \{T_2 = 11, C_2 = 3\}$ and $\mathbf{t}_3 = \{T_3 = 15, C_3 = 5\}$ as a “counter example” to show some “mistakes” in **theorem 4**. Let's clarify the **theorem 4** and the “counter example” to have better understanding of both. Consider the task $\mathbf{t}_1 = \{T_1 = 8, C_1\}, \mathbf{t}_2 = \{T_2 = 11, C_2\}$ and $\mathbf{t}_3 = \{T_3 = 15, C_3\}$, with the same T_1, T_2, T_3 used in the “counter example” of Devillers and Goossens⁴ and all feasible C_1, C_2, C_3 combinations. The processor utilization factor for all feasible C_1, C_2, C_3 combinations are presented in Table 1 and Figure 6. The C_1 values are in rows, the C_2 values are in columns, and the C_3 values are calculated to fully utilize the processor.

Figure 5 - Full utilization for a task set with $m = 7$.

Table 1 – Full processor utilization factor for all feasible C_1, C_2, C_3 combinations.

$C_2 \setminus C_1$	1	2	3	4	5	6
1	1,083	0,941	0,933	0,924	0,916	0,908
2	0,907	0,898	0,890	0,882	0,873	-
3	0,864	0,856	0,848	0,839	-	-
4	0,822	0,814	0,805	-	-	-
5	0,846	0,838	-	-	-	-
6	0,870	0,862	-	-	-	-
7	0,895	-	-	-	-	-
8	0,919	-	-	-	-	-

Figure 6 – Utilization factor $U(C_1, C_2)$ with C_3 chosen to fully utilize the processor.

Figure 7 – Task set for minimum U and fully utilization.

The minimum U is $U_{\text{lub}} = 0.805$ at task $\mathbf{t}_1, \mathbf{t}_2, \mathbf{t}_3$,
 $\mathbf{t}_1 = \{T_1 = 8, C_1^{\text{lub}} = 3\}$, $\mathbf{t}_2 = \{T_2 = 11, C_2^{\text{lub}} = 4\}$
and $\mathbf{t}_3 = \{T_3 = 15, C_3^{\text{lub}} = 1\}$. This task set is
presented in Figure 7.

So the “counter example” of Devillers and Goossens⁴
is wrong because in order to disprove **theorem 4** such
“counter example” must satisfy the hypothesis of
theorem 4. But we cannot choose the combination
 $\mathbf{t}_1 = \{T_1 = 8, C_1 = 2\}$, $\mathbf{t}_2 = \{T_2 = 11, C_2 = 3\}$
and $\mathbf{t}_3 = \{T_3 = 15, C_3 = 5\}$ used in Devillers and
Goossens⁴ because it fully utilizes the processor and
provides $U = 0.856$, **but** it does not minimize the
processor utilization factor. **Theorem 4** says that we
must use the task $\{\mathbf{t}_1 = \{T_1 = 8, C_1^{\text{lub}} = 3\}$,
 $\mathbf{t}_2 = \{T_2 = 11, C_2^{\text{lub}} = 4\}$ $\mathbf{t}_3 = \{T_3 = 15, C_3^{\text{lub}} = 1\}\}$ that
minimizes the processor utilization factor among all
 C_1, C_2, C_3 combinations that satisfies full utilization
only.

Liu and Layland³ generalized the **theorem 4** as
theorem 5: *For a set of m tasks with fixed priority
order, the least upper bound to the processor utilization
factor is $U_{\text{lub}(m)}^m = m \cdot (2^{1/m} - 1)$.*

Proof: to do that they proved that for any task set that
fully utilize the processor, the processor utilization
factor U' is less than or equal U_{lub}^m ($U' \leq U_{\text{lub}}^m$) and,
consequently, to determine the least upper bound of the
processor utilization factor we need only consider task
sets in which the ratio between any two request periods
is less than two.

MISUNDERSTANDINGS

In the specific literature, as in Burns and Wellings⁵,
page 347, we see simpler forms of Liu and Layland³

equations and conclusions for schedulability: “for a
single processor system, a set of m independent,
periodic processes with only execution-time deadlines
is schedulable if the overall processor utilization is less
than $m \cdot (2^{1/m} - 1)$. For a large m this asymptotically
approaches 0.693. Therefore, all processes sets are
schedulable if processor utilization is less than 69%”.

(M1) Looking quickly, at first impression, we can
wrongly think that if the processor utilization factor is
greater than $m \cdot (2^{1/m} - 1)$ (for m finite) or than 69,3 %
($m \rightarrow \infty$) the task set is not feasible using RMS. This

result assumes that all task periods $T_1, T_2, \dots, T_m$ are
mutually prime, the worst possible relationship among
them. If there is more favorable relationship among
them the system can achieve higher utilization factor. In
some special cases, as one shown in Burns and
Wellings⁵, p. 347, and reproduced in Figure 8,
 $\mathbf{t}_1 = \{T_1 = 80, C_1 = 40\}$, $\mathbf{t}_2 = \{T_2 = 40, C_2 = 10\}$
and $\mathbf{t}_3 = \{T_3 = 20, C_3 = 5\}$, U reaches 100%.

(M2) On the other hand, we can wrongly think, if using
RMS and the processor utilization time is less than 69,3
% ($m \rightarrow \infty$) we have available time to add tasks to
the processor because for large m the processor
utilization factor asymptotically approaches 0.693. We
already saw it as a specification item in a simulation
contract. More than that, we faced a real problem where
there was available time in the processor and we could
add nothing. In fact, for a task that fully utilizes the
processor if at least one time unit is added, it will be
infeasible even there is available processor time. In this
situation we need evaluate a new scheduler or, in worst
case, replace the processor with a faster one.

Figure 8 – Task set for $\mathbf{t}_1 = \{T_1 = 80, C_1 = 40\}$, $\mathbf{t}_2 = \{T_2 = 40, C_2 = 10\}$ and $\mathbf{t}_3 = \{T_3 = 20, C_3 = 5\}$.

RATE MONOTONIC SCHEDULER (RMS) RESTRICTIONS

In the previous sections we demonstrated that using RMS all task sets are schedulable if the processor utilization factor is less than 69,3% using worst possible relationship (mutually prime) among all task periods. We can improve it choosing $T_1, T_2, \dots, T_m$ with good relationships among (T_i, T_j) . As shown in the previous example, the simplest way to do achieve 100% processor utilization factor using RMS is to make null the fractional part of T_m / T_i , $\{T_m / T_i\} = 0$, for $i = 1, 2, \dots, m-1$. It is good, but not enough, because in the real world this can no be always done and because it still necessary to consider the switching costs between tasks.

FIRST DEADLINE FIRST SCHEDULER (FDFS) OR EARLIEST DEADLINE FIRST (EDF)

In the second part of the paper Liu and Layland³ proved that FDFS or EDF scheduler can schedule any tasks that are schedulable by any other method; and the least upper bound on the processor utilization factor is uniformly 100%. Liu and Layland³ proposed and proved the **theorem 7** (their number): *for a given set of m tasks, the deadline driven scheduling algorithm is feasible if and only if*

$$(C_1 / T_1) + (C_2 / T_2) + \dots + (C_m / T_m) \leq 1. \quad (11)$$

A FDFS graphical representation for the task $t_1 = \{T_1 = 8, C_1 = 3\}$, $t_2 = \{T_2 = 11, C_2 = 4\}$ and $t_3 = \{T_3 = 15, C_3 = 1\}$ used in theorem 4 is presented

Figure 9– FDFS for task $t_1 = \{T_1 = 8, C_1 = 3\}$, $t_2 = \{T_2 = 11, C_2 = 4\}$ and $t_3 = \{T_3 = 15, C_3 = 1\}$.

in Figure 9.

True, it improves the RMS; but in real world it is a complex implementation and it does not solve the problem of switching costs between tasks.

IMPROVEMENTS ON THE RMS AND FDF SCHEDULERS

Even though the Rate Monotonic Scheduler and the First Deadline First Scheduler are based in solid theory in scheduling area, they are not suitable for a complex and dynamic simulation environment with great number of players (big number of tasks) mostly because they do not solve the problem of switching costs among tasks and can guarantee the service only for a known number of tasks. To overcome these problems it is necessary and possible to do some extensions of those algorithms in order to apply them to an environment that always changes like: an operation flight program for a fighter in a complex an

dynamic defense system; a spaceship or spacecraft in an interplanetary trip flying through debris; or a future microcomputer inside human body as a complementary part of its defense or behavior systems. This will be presented and detailed in a future paper.

CONCLUSIONS

In this work we discussed and clarified that the rate monotonic scheduler and the first deadline first scheduler are solid theory for real time simulation and control. These algorithms are largely know and discussed in the real time literature; but in this work: 1) we discussed and clarified some aspects of their original propositions and we improved such

propositions by explicitly writing down some of their incomplete or even hidden passages and transformations; 2) we also discussed and clarified some mistakes done in those counter-examples and critiques, by replacing them with new examples that confirm the algorithms and illustrate the points discussed and clarified.

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